APPENDIX 5B. SCALING RELATIONSHIPS IN TRANSFORMER MANUFACTURING

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

APPENDIX 5B. SCALING RELATIONSHIPS IN TRANSFORMER MANUFACTURING

5B.1 INTRODUCTION

There exist certain fundamental relationships between the ratings in kilovolt-amperes (kVA) of transformers and their physical size and performance. A rather obvious such relationship is the fact that large transformers of the same voltage have lower percentage losses than small units, i.e., large transformers are more efficient. These size-versus-performance relationships arise from fundamental equations describing a transformer's voltage and kVA rating. For example, by fixing the kVA rating and voltage frequency, the product of the conductor current density, core flux density, core cross-sectional area, and total conductor cross-sectional area is constant.

To illustrate this point, consider a transformer with frequency, magnetic flux density, current density, and basic impulse insulation levels (BIL) all fixed. If one enlarges (or decreases) the kVA rating, then the only free parameters are the core cross-section and the core window area through which the windings pass. Thus, to increase (or decrease) the kVA rating, the dimensions for height, width, and depth of the core/coil assembly may be scaled equally in all directions. Careful examination reveals that linear dimensions vary as the ratio of kVA ratings to the ¼ power. Similarly, areas vary as the ratios of kVA ratings to the ½ power and volumes vary as the ratio of the kVA ratings to the ¾ or 0.75 power. Hence the term "0.75 scaling rule." Table 5B.1 depicts the most common scaling relationships in transformers.

Table 5B.1 Common Scaling Relationships in Transformers

Parameter Being Scaled	Relationship to kVA Rating (varies with ratio of kVA^x)
Weight	$(kVA_1/kVA_0)^{3/4}$
Cost	$(kVA_1/kVA_0)^{3/4}$
Length	$(kVA_1/kVA_0)^{1/4}$
Width	$(kVA_1/kVA_0)^{1/4}$
Height	$(kVA_1/kVA_0)^{1/4}$
Total Losses	$(kVA_1/kVA_0)^{3/4}$
No-load Losses	$(kVA_1/kVA_0)^{3/4}$
Exciting Current	$(kVA_1/kVA_0)^{3/4}$
% Total Loss	$(kVA_1/kVA_0)^{-1/4}$
% No Load Loss	$(kVA_1/kVA_0)^{-1/4}$
% Exciting Current	$(kVA_1/kVA_0)^{-1/4}$
% Resistance (R)	$(kVA_1/kVA_0)^{-1/4}$
% Reactance (X)	$(kVA_1/kVA_0)^{1/4}$
Volts/Turn	$(kVA_1/kVA_0)^{1/2}$

The three elements listed below are true as the kVA rating increases or decreases, if the following factors are held constant: the type of transformer (distribution or power transformer, liquid-filled or dry-type, single-phase or three-phase), the primary voltage, the core configuration, the core material, the core flux density, and the current density (amperes per square inch of conductor cross-section) in both the primary and secondary windings.

1. The physical proportions are constant (same relative shape),
2. The eddy loss proportion is essentially constant, and
3. The insulation space factor (voltage or BIL) is constant.

In practical applications, it is rare to find that all of the above are constant over even limited ranges; however, over a range of one order of magnitude in both directions (e.g., from 50kVA to 5kVA or from 50kVA to 500kVA), the scaling rules shown in Table 5B.1 can be used to establish reasonable estimates of performance, dimensions, costs, and losses. In practice, these rules can be applied over even wider ranges to estimate general performance levels. The same quantities are depicted graphically in Figure 5B.1 for reference.

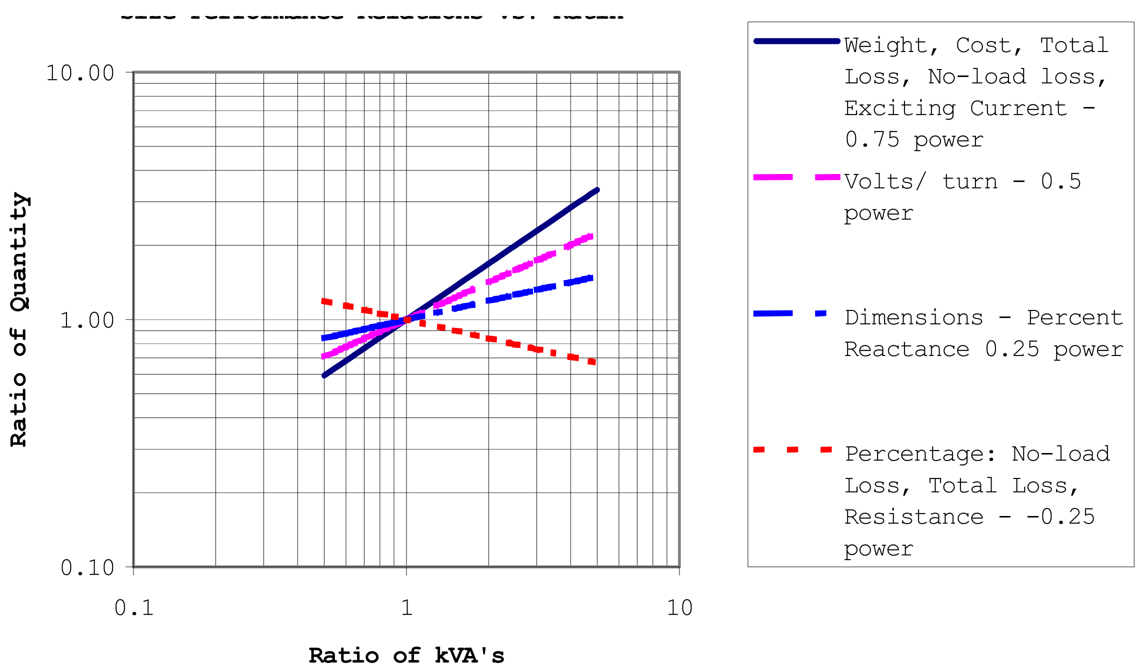

Figure 5B.1 Size and Performance Relationships by kVA Rating

To illustrate how the scaling laws are used, consider two transformers with kVA ratings of S_0 and S_1. The no-load losses (NL) and total losses (TL) of these two transformers would be depicted as NL_0 and TL_0, and NL_1 and TL_1. Then the relationships between the NL and TL of the two transformers could be shown as follows:

$$NL_1 = NL_0 \left(\frac{S_1}{S_0}\right)^{0.75} \quad \text{and} \quad TL_1 = TL_0 \left(\frac{S_1}{S_0}\right)^{0.75}$$

These two equations can be manipulated algebraically to show that the load loss also varies to the 0.75 power. Starting with the concept that total losses equals no-load losses plus load losses, one can derive the relationship for load loss (LL), and show that it also scales to the 0.75 power. Specifically:

$$LL_1 = TL_1 - NL_1$$

Plugging the TL1 and NL1 terms into this equation:

$$LL_1 = TL_0 \left(\frac{S_1}{S_0}\right)^{0.75} - NL_0 \left(\frac{S_1}{S_0}\right)^{0.75}$$

$$= \left(TL_0 - NL_0\right)\left(\frac{S_1}{S_0}\right)^{0.75}$$

That is,

$$LL_1 = LL_0\left(\frac{S_1}{S_0}\right)^{0.75}$$

In this way, the 0.75 scaling rule can be used to derive the losses of a transformer, knowing the losses of a reference unit, if the specified type of transformer is held constant, and key parameters are fixed—such as the type of core material, core flux density, and conductor current density in the high and low voltage windings.

5B.2 THEORY AND BASIS FOR SCALING RULES

To understand the origins of winding and output coefficients and related scaling laws, it is necessary to review some basic equations and definitions. Most are lifted freely or derived from similar material in *Modern Power Transformer Practice*, Wiley 1979, edited by R. Feinberg.[1] No mathematics beyond elementary algebra is required, but a good deal of implied physics and electrical engineering is required to fully appreciate these derivations.

5B.2.1 Power and Voltage Equations

The machine equation relates the induced volts, V, per phase to the number of turns (N) the frequency (f) in Hertz, the peak core flux density B_m in Tesla, and the cross-sectional area of the core steel (A_{Fe}) in square meters. The units are mixed to simplify the basic equations, a common practice in transformer design texts. The machine equation is derived from Faraday's law, which is expressed as

$$v = -N\frac{\partial\phi}{\partial t}$$

where v is the instantaneous value of V, and $\frac{\partial\phi}{\partial t}$ is the derivative of changing magnetic flux with respect to time.

Considering V as the root-mean-square (RMS) value of a sine-wave alternating current voltage, the above equation can be converted into:

$$V/N = 4.44\, f\, B_m\, A_{Fe} \qquad \textbf{Eq. 1}$$

The voltage and turns may apply to either the primary or the secondary winding and, for the ideal transformer with no losses and no-leakage flux,

$$V_1/V_2 = N_1/N_2 = n = I_2/I_1$$

where V_1 and V_2 represent primary and secondary voltages respectively, N_1 and N_2 primary and secondary turns, and I_1 and I_2 primary and secondary currents in amperes (amps). The quantity *n* is referred to as the "turns ratio." With the parameters defined, and using Equation 1, the output or transformer capacity (S) in megavolt-amperes (MVA) per phase can be expressed as:

$$S = 4.44\ f\ B_m\ A_{Fe}\ N\ I \qquad \textbf{Eq. 1a}$$

The overall cross-section of primary plus secondary conductors in square meters is

$$A_{Cu} = (N_1a_1 + N_2a_2) \times 10^{-6}$$

and, assuming current densities for primary and secondary windings to be equal, then

$$A_{Cu} = 2 \times 10^{-6}\ Na$$

where "a" is the conductor cross-section in square millimeters (mm^2) of an individual turn referred to the winding with N turns, and a_1 and a_2 are conductor cross-sections of primary and secondary turns, respectively. As long as the winding current densities are equal, either winding may be used as reference, provided the choice of primary or secondary is consistent. Starting with Equation 1a, using the A_{Cu} relationship explained above, and letting J represent current density in amps per mm^2:

$$S = 2.22\ f\ B_m\ J\ A_{Fe}\ A_{Cu} \qquad \textbf{Eq. 2}$$

Let A_w be the core window area in square meters, and k_w the window space factor, as given by $2\ A_{Cu}/A_w$. (Refer to Figure 5B.2 and note that, in a three-phase transformer, there are two coil phases occupying a given core window). This fraction is indicative of the insulation and cooling channel requirements. For distribution transformers, k_w is found to be about 0.3–0.4 for nominal 12 kV systems. Using these definitions,

$$S = 1.11\ f\ B_m\ J\ A_{Fe}\ k_w\ A_w \qquad \textbf{Eq. 2a}$$

Note that, for a given MVA rating, and specified flux and current densities, the product of conductor and core cross-section is constant and inversely related; i.e. $A_{Fe} \propto 1/A_{Cu}$.

5B.2.2 Losses

Ideally, if the values of energy loss in Watts per kilogram (W/kg) of unit mass of the core and windings are known, the total core and load losses (P_{Fe} and P_{Cu}) can be readily obtained.

These results are accomplished by multiplying the W/kg for both core and windings by the core mass and the conductor mass respectively (or by their volumes times material densities).

The Department uses the convention that lower case corresponds to per-unit quantities and upper case corresponds to total or total-per-phase quantities. Load losses consist of resistive (p_R) and eddy (p_i) components. Expressions can be derived that express each in terms of the conductor properties and geometry. The fraction of eddy losses plays an important role and can be expressed as

$$\%P_i = 100\, P_i/P_R, \text{ or } \; P_i = P_R\left(\frac{\%P_i}{100}\right)$$

Ignoring stray loss, (which is associated with eddy losses), let P_t represent total load loss for a three-phase transformer. That is,

$$P_t = 3P_{Cu}$$

Also assume the same eddy loss fraction in primary and secondary windings.

$$P_{Cu} = P_R + P_i = P_R + P_R\left(\frac{\%P_i}{100}\right) = \left(1+\frac{\%P_i}{100}\right)P_R = k_i P_R$$

Closely associated with the load loss of a transformer is its impedance. When the load loss of a given transformer is determined by test (the watt-meter reading in the test circuit), that same test also provides the value of the impedance (the voltmeter reading in the test circuit). Impedance in a transformer is expressed in terms of the "impedance voltage," which is defined as "the voltage required to circulate rated current through one of two specified windings of a transformer when the other winding is short-circuited, with the windings connected as for rated voltage operation" (IEEE C57.12.80).

For convenience, "percent impedance," %Z, is used to describe the impedance voltage of a transformer. In accordance with the definition given above,

$$\%Z = \frac{IZ \times 100}{V}$$

that is, when related to the primary or secondary winding of a transformer, the percent impedance is the percent voltage drop due to impedance when rated current flows through the respective primary or secondary winding of the transformer.

The %Z may be represented by its resistive and reactive components, %R and %X, as

$$\%Z = \sqrt{(\%R)^2 + (\%X)^2}$$

Therefore, one can express percent resistance (%R) as follows:

$$\%R = \frac{IR \times 10^2}{V}$$

Note that R in the numerator must represent the total resistance in the transformer windings. Therefore, if the transformer is being viewed from the primary terminals, the value of R would be the total resistance of the primary winding, plus the total resistance of the secondary winding referred to the primary winding, $(R_2(N_1/N_2)^2)$.

Where the percent impedance, percent reactance, and percent resistance are related to the voltage across the primary or secondary winding of a transformer, the percent load loss ($\%I^2R$) is related to the MVA capacity of the transformer, stray loss being ignored as stated previously.

Multiplying numerator and denominator in the above equation by I, and letting P_t represent total load loss in watts and S represent the MVA per phase rating, one can determine the percent load loss as:

$$\text{Percent load loss} = \frac{I^2R \times 10^2}{I \times V} = \frac{I^2R \times 10^2}{3S \times 10^6}$$

$$\therefore \%R = \frac{10^{-4} P_t}{3S}$$

Thus, an expression of %R is equivalent to indicating the transformer's load loss.

From Equation 2, it is evident that, once the core flux density and current density are fixed, the transformer rating is dependent on the core cross-section and window area. Next, one can derive information about the window shape.

In a detailed discussion of the reactance, the electrical characteristics would depend on:

- the ratio of winding height (h) to the winding mean turn (s), and
- the ratio of the cross-sectional areas of the core and conductor (A_{Fe} / A_{Cu}).

The mean value of s (a linear measurement, recording the circumference), is given by the equation $s = (s_1 + s_2)/2$, where s_1 is the mean turn of the primary winding and s_2 is the mean turn of the secondary winding.

These ratios, together with the necessary space factors for insulating and cooling clearances, establish the relative volumes of the core and conductor. Consequently, if fixed values for the specific loadings and, therefore specific losses for core and conductor can be assumed, the ratios of core loss and load loss are established.

The following application of relationships derives an expression relating the flux and current densities. The expression starts with:

$$P_{Cu} = \left(1 + \frac{\%P_i}{100}\right)P_R = k_i P_R$$

$$P_{Cu} = (I_1^2 R_1 + I_2^2 R_2)k_i,$$

where subscripts 1 and 2 indicate primary and secondary windings, respectively. The resistance per phase of the primary winding is given by

$$R_1 = \frac{\rho N_1 s_1}{a_1} \text{ohms},$$

where a_1 is the cross-sectional area of the primary copper conductor, and ρ is the resistivity at full load operating temperature of the conductor, 21.4 x 10^{-3} ohm-meters. The value of R_2 is similarly obtained:

$$\therefore P_{Cu} = \left(\frac{I_1^2 \rho N_1 s_1}{a_1} + \frac{I_2^2 \rho N_2 s_2}{a_2}\right)k_i$$

$$\therefore P_{Cu} = IN\left(\frac{I_1 s_1}{a_1} + \frac{I_2 s_2}{a_2}\right)\rho k_i$$

where IN is the ampere-turns in either winding. As before, the assumption of equal current densities in the windings is made, driven by the condition for minimum I^2R loss. Accordingly,

$$P_{Cu} = 2INJs\rho\, k_i$$

$$\therefore J = \frac{P_{Cu}}{2INs\rho k_i}$$, the current density equation.

Multiplying Equation 1 by I and rearranging algebraically, one gets:

$$IN = \frac{VI}{4.44 f B_m A_{Fe}}$$

It was established earlier that S is the rating per phase in MVA, i.e., $VI = 10^6 S$. Thus:

$$\therefore IN = \frac{10^6 S}{4.44 f B_m A_{Fe}}$$

Using the current density equation, substituting the resistivity value for ρ, and the above value for IN, one can derive that:

$$J = \frac{104 \times 10^{-6} f B_m A_{Fe} P_{Cu}}{k_i s S}$$

The watts of conductor loss (for copper) can be expressed as a percentage of the transformer MVA rating:

$$\%P_{Cu} = \frac{P_{Cu} \times 10^2}{S}$$

or, in kilowatts:

$$\%P_{Cu} = \frac{P_{Cu} \times 10^2}{S \times 10^3} = \frac{0.1 P_{Cu}}{S}.$$

By substituting in the revised equation for J (amperes per square meter), one gets

$$J = \frac{104 \times 10^{-6} f B_m A_{Fe} S}{k_i s S} \times \frac{\%P_{Cu}}{0.1} = \frac{1040 \times 10^{-6} f B_m A_{Fe}}{k_i s} \times \%P_{Cu} \qquad \textbf{Eq. 3}$$

If aluminum windings were used instead of copper, a value of 655 would be substituted for 1040. The expression assumes equal J in both windings, and that both windings are made of the same material. The losses are expressed at operating temperature.

If J and B_m are chosen independently, the transformer will have a natural value of conductor loss depending on the ratio A_{Fe}/s. Conversely, if losses are specified, the choice of J is determined by B_m and A_{Fe}/s. Note that this relationship gives no information about the other transformer dimensions. The impedance, voltage, and other space requirements provide the majority of this information.

5B.2.3 Output and Winding Coefficients

Starting with the output or power Equation 2, one can write:

$$S = 2.22\, f\, B_m\, J\, A_{Fe}\, A_{Cu} \quad \text{or} \quad A_{Fe} = \frac{S}{2.22\, f\, B_m J\, A_{Cu}}$$

Then, without changing the value, one can state:

$$A_{Fe} = \sqrt{\frac{S^2}{\left(2.22 f\, B_m J\, A_{Cu}\right)^2}} = \sqrt{S}\sqrt{\frac{2.22 f\, B_m J\, (A_{Fe})(A_{Cu})}{\left(2.22 f\, B_m J\, A_{Cu}\right)^2}} \quad \text{or}$$

$$A_{Fe} = \sqrt{S}\sqrt{\frac{A_{Fe}}{(2.22 f\, B_m J)(A_{Cu})}} \qquad \textbf{Eq. 4}$$

Use K_{AS} to represent the portion of Equation 4 to the right of $\sqrt{S}$

The expression K_{AS} is essentially constant for a wide range of transformer classes and is called the output coefficient. For three-phase, liquid-filled distribution transformers at 60 Hz, the value of K_{AS} ranges from 0.050 to 0.055, with a nominal median value of 0.052. For single-phase, wound-core, liquid-filled units at 60 Hz, the median value is about 0.040.

In a similar fashion, making use of Equation 4, we can restate Equation 1 as follows:

$$\frac{V}{N} = 4.44 f B_m A_{Fe} = \sqrt{\frac{(4.44 f B_m)^2 S A_{Fe}}{2.22 f B_m J A_{Cu}}}$$

$$= \sqrt{\left(\frac{8.88 f B_m}{J}\right)\left(\frac{A_{Fe}}{A_{Cu}}\right)(S)} = K_{VS}\sqrt{S} \qquad \textbf{Eq. 5}$$

The expression K_{VS} is also essentially constant for a wide range of transformer classes and is called the winding coefficient. One can also express K_{VS} in terms of K_{AS}:

$$K_{VS} = 4.44\, f\, B_m\, K_{AS}$$

For 60 Hz systems, this may be rewritten as $K_{VS} = 266.4\ B_m\ K_{AS}$. Thus the median values for K_{VS} become 21.5 for three-phase and 17.0 for single-phase, wound-core distribution transformers at 60 Hz with $B_m = 1.55$ Tesla. Equations 4–5 provide initial estimates for transformer dimensions in studies. They are the starting basis for the scaling laws used to scale designs and performance. Typical values are given in Table 5B.2 for core type, liquid-filled, 60 Hz distribution transformers at 12 kV, 95 kV BIL.

Table 5B.2 Nominal 60 Hz, Core-Type, Liquid-Filled, 12 kV Distribution Transformers

Class of Dist.	**J(A/mm²)**		**B_m (Tesla)**	**A_{Fe}/A_{Cu}**		**K_{AS}**		**K_{VS}**	**%X**
Transf.	Range	Nominal	Nominal	Range	Nominal	Range	Nominal		
3-Phase	2.4-3.2	2.7	1.55	1.4-2.8	1.6	0.050-0.055	0.052	21.5	4.75
1-Phase	2.0-2.5	2.3	1.55	0.65-0.85	0.8	0.038-0.043	0.041	17.0	4.75

5B.2.4 Scaling Laws

Having established the output and winding coefficients, it is instructive to examine the origin of the 0.75 rules for scaling transformer losses. To illustrate, first of all, one needs to set relationships as follows:

$$\frac{V}{N} = K_{VS}\sqrt{S}$$

$$A_{Fe} = K_{AS}\sqrt{S}$$

$$A_{Cu} = K_{CS}\sqrt{S} \quad \text{, (where} \quad K_{CS} = \frac{1}{K_{AS}} \text{)}$$

$$s \sim (A_{Fe}^{0.5} + \frac{b_w}{4}) \sim S^{0.25}$$

The shape of the window is set by voltage and the ratio h/s, which is essentially constant for a given voltage and size, thus setting b_w. Refer to Figure 5B.2 for dimensional definitions.

Now, one considers the load losses, P_{Cu} (in kW/phase):

$$P_{Cu} = \frac{I^2 R}{1000} = \left(\frac{S}{V}\right)^2 \frac{R}{1000} = \left(\frac{S}{V}\right)^2 \frac{R}{1000}$$

$$= \frac{4.28\times10^{-17} S^2 s N^2}{A_{Cu} V^2} = K\sqrt{S}\times s = K'S^{0.75}$$

The other scaling laws are derived in a similar fashion.

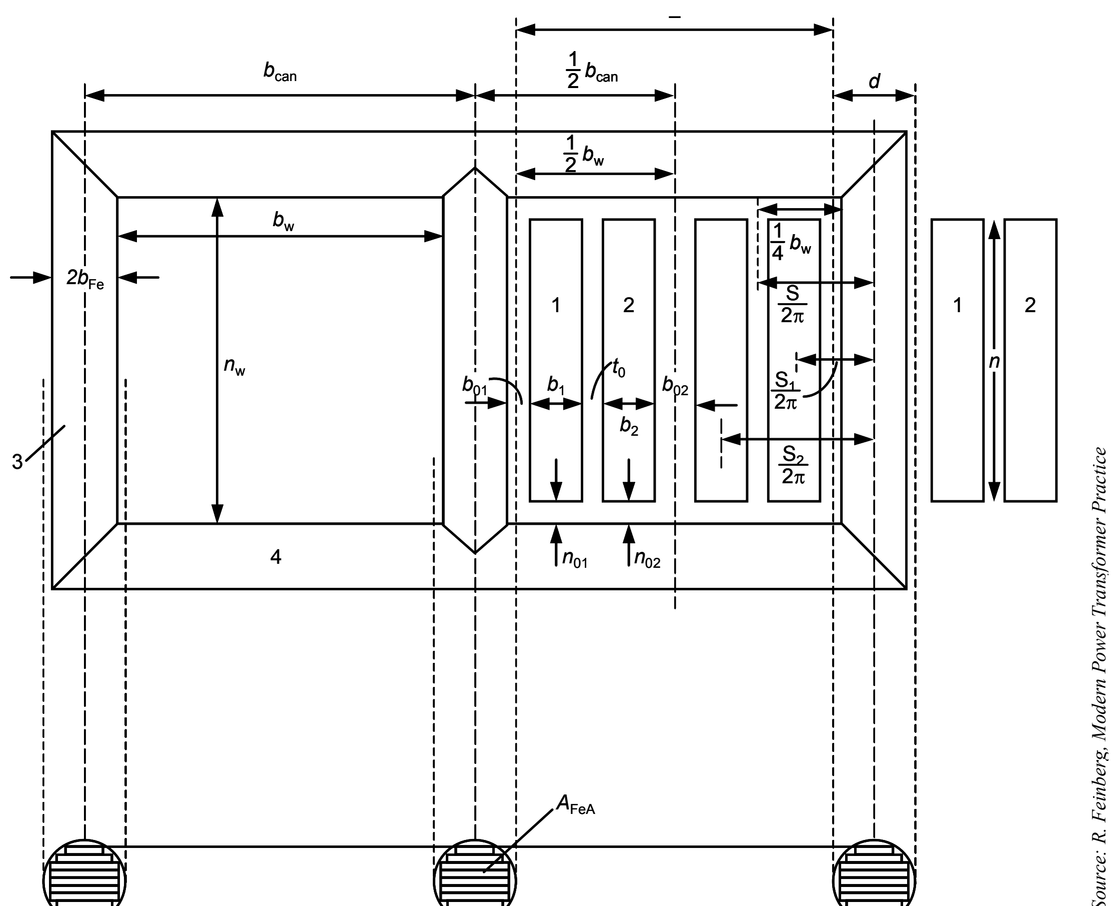

Figure 5B.2 Basic Three-Phase Transformer Dimensions

REFERENCES

1. Modern power transformer practice, Edited by R Feinberg, Wiley Publishers, New York, NY, 1979. ISBN: 047026344X