

Glass-like thermal conductivity in high efficiency thermoelectric materials

Eric S. Toberer

Physics Dept.

Materials & Metallurgical Eng. Dept.

Colorado School of Mines

Jeff Snyder

Materials Science

Caltech



Funding:

Beckman Foundation

NASA-JPL

ARPA-E

DARPA

ZT Plus

BSST

Outline

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

v_g - phonon group velocity
 τ - phonon relaxation time

Group velocity (v_g)

- Structural complexity
 - $\text{Yb}_{14}\text{MnSb}_{11}$
 - $\text{Ca}_5\text{Al}_2\text{Sb}_6$
 - Ca_3AlSb_3

Scattering (τ)

- Intrinsic point defects
 - $\text{La}_{3-x}\text{Te}_4$
 - Zn_4Sb_3
- Nanostructures
 - Zn_4Sb_3
 - PbTe

Designing low κ_L materials - group velocity

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

v_g - phonon group velocity
 τ - phonon relaxation time



$$v_s = \sqrt{\frac{B}{d}}$$

v_s - acoustic speed of sound
 B - elastic modulus
 d - density

New materials should have 'soft' bonding
and be composed of heavy atoms

Designing low κ_L materials - group velocity

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

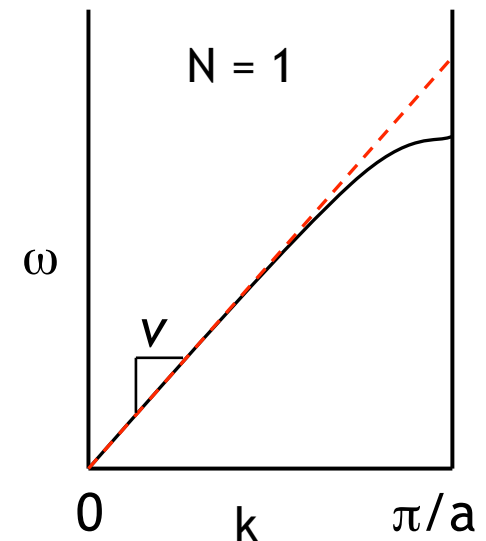
v_g - phonon group velocity
 τ - phonon relaxation time

$$v_s = \sqrt{\frac{B}{d}}$$

v_s - acoustic speed of sound
 B - elastic modulus
 d - density

$$v_g(\omega) = v_s$$

Valid only for phonons at long wavelengths!



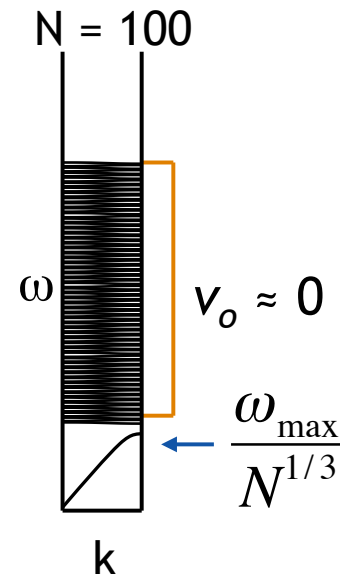
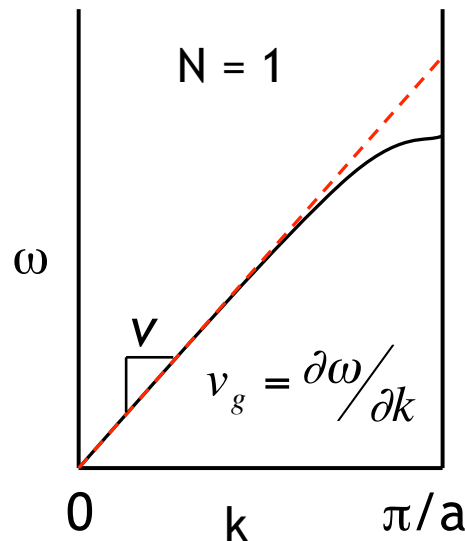
$$v_g = \frac{\partial \omega}{\partial k}$$

Lattice thermal conductivity prediction

In complex materials:

3 acoustic branches (2 transverse, 1 longitudinal)

$3(N-1)$ optical branches, where N is # atoms in primitive cell

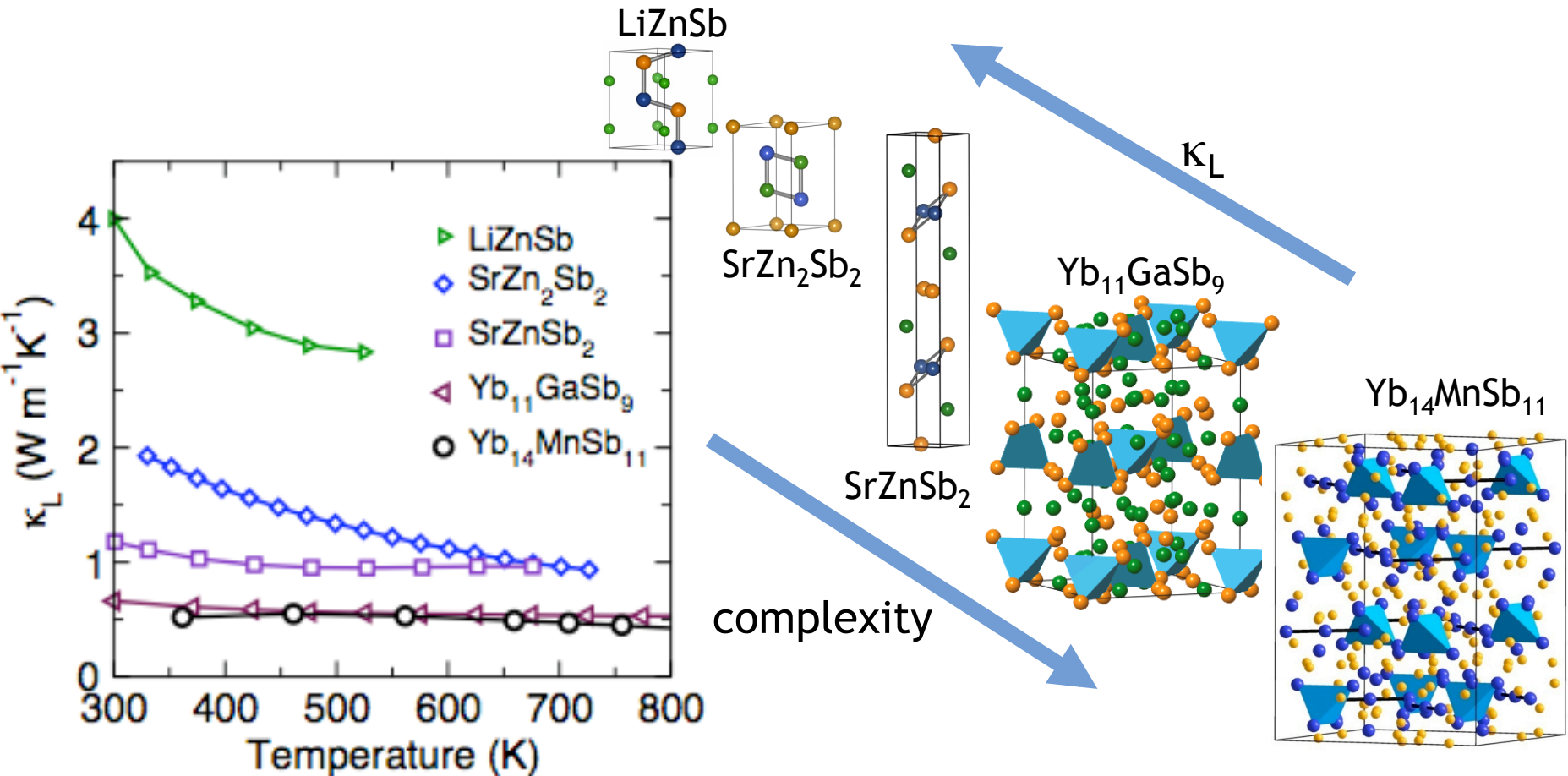


$$\kappa_a = \frac{c\bar{M}\theta_D^3 a}{N^{1/3}\gamma^2 T}$$

True in practice?

Antimonides and thermal conductivity

Primitive unit cell complexity good indicator for κ_L



Yi et al *Chem Mater* (2010)

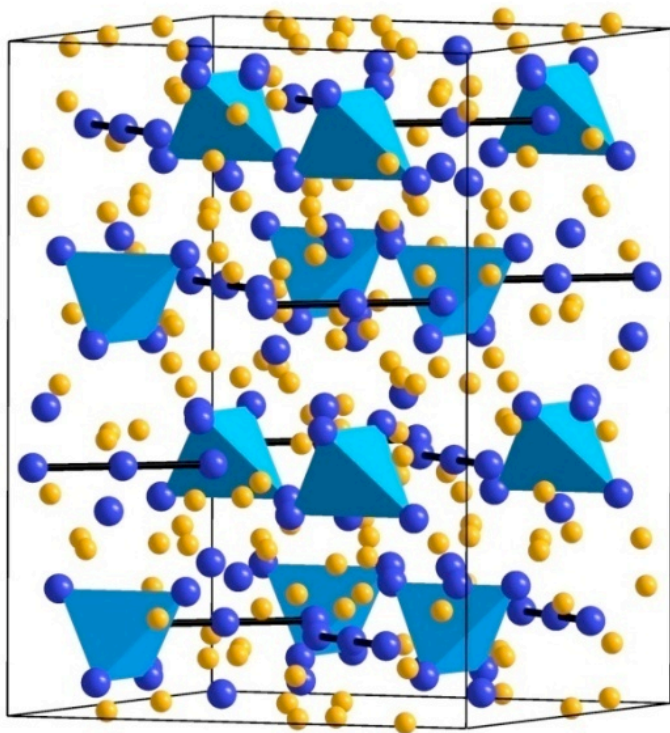
Toberer et al *Dalton Trans* (2010)

May et al *J. Appl. Phys* (2009)

Toberer et al *J. Appl. Phys.* (2009)

Toberer et al *Adv. Funct. Mater.* (2008)

$\text{Yb}_{14}\text{Mn}_x\text{Al}_{1-x}\text{Sb}_{11}$ - Glass like thermal conductivity



Low κ_L arises from:

Complexity ($N = 104$)

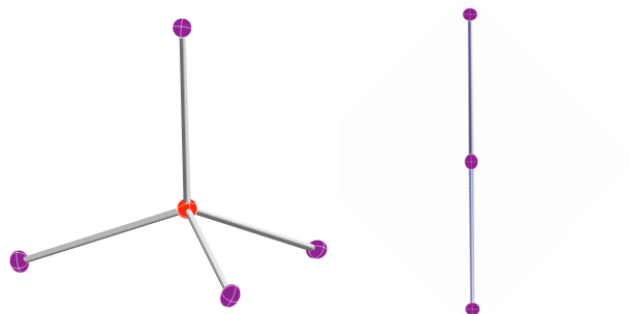
Density

Mass contrast

Anharmonicity ($\gamma = 1.5$)

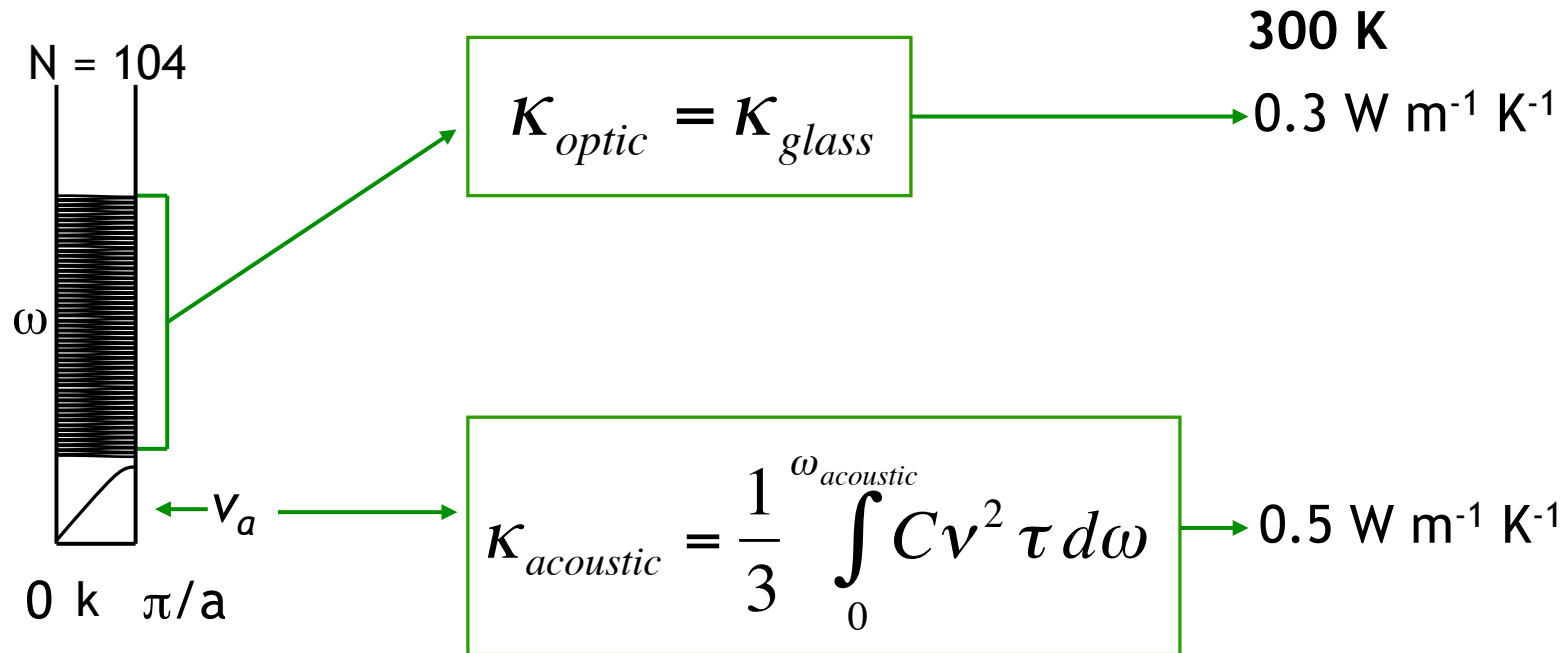
No evidence of rattling:

Single crystal XRD thermal ellipsoids



Understanding thermal conductivity in $\text{Yb}_{14}\text{MnSb}_{11}$

v_{optic} so low, we will use the theory developed for glassy materials.



$$K_{\text{lattice}} = K_{\text{acoustic}} + K_{\text{optic}}$$

300K

Prediction: $0.8 \text{ W m}^{-1} \text{ K}^{-1}$

Reality: $0.6 \text{ W m}^{-1} \text{ K}^{-1}$

Zintl $\text{Yb}_{14}\text{Mn}_x\text{Al}_{1-x}\text{Sb}_{11}$ - Next generation power source

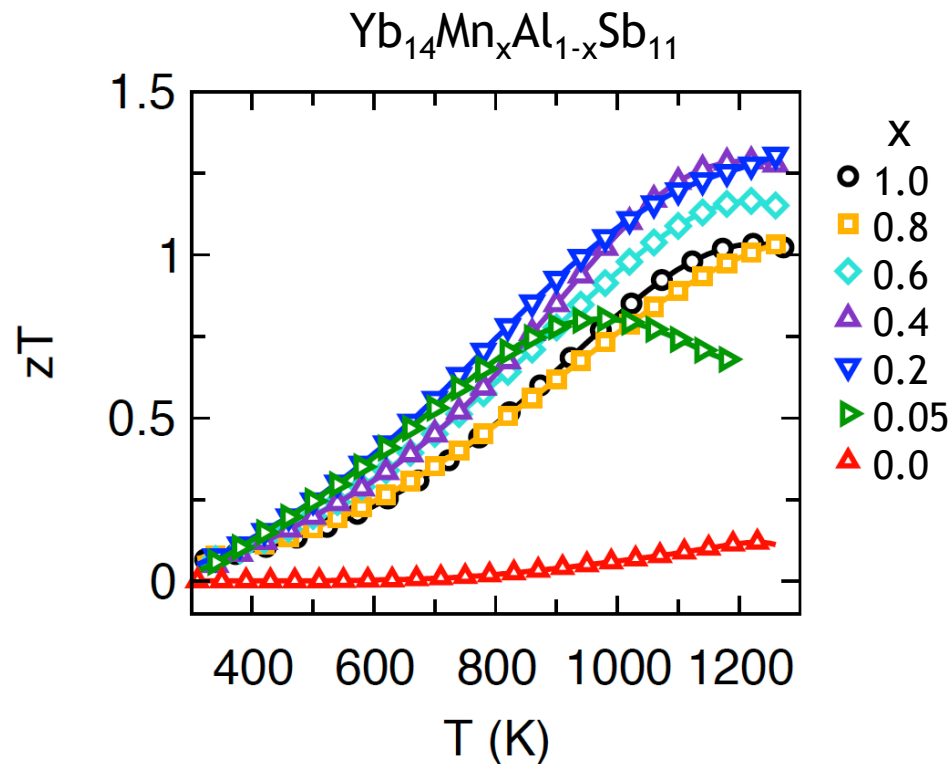


Figure of merit (zT) > 1 at 1200K!

Factors contributing to good thermoelectric performance:

- Band gap (0.5 eV) and moderate mobility
- robust carrier concentration control
- glass-like κ_L

Rauscher, et al, *Chem. Mater* (2010)
Cox, et al, *Chem. Mater.* (2009)
Toberer et al *Appl. Phys. Lett.* (2008)
Brown et al, *Chem. Mater.* (2008)
Toberer et al, *Adv. Funct. Mater.* (2008)
Brown et al *Chem. Mater.* (2006)

Zintl $\text{Yb}_{14}\text{Mn}_x\text{Al}_{1-x}\text{Sb}_{11}$ - Next generation power source

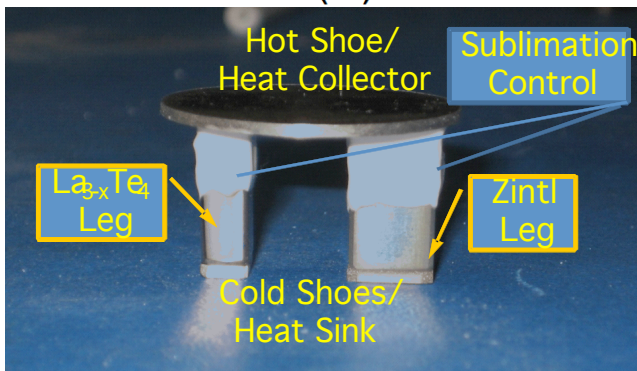
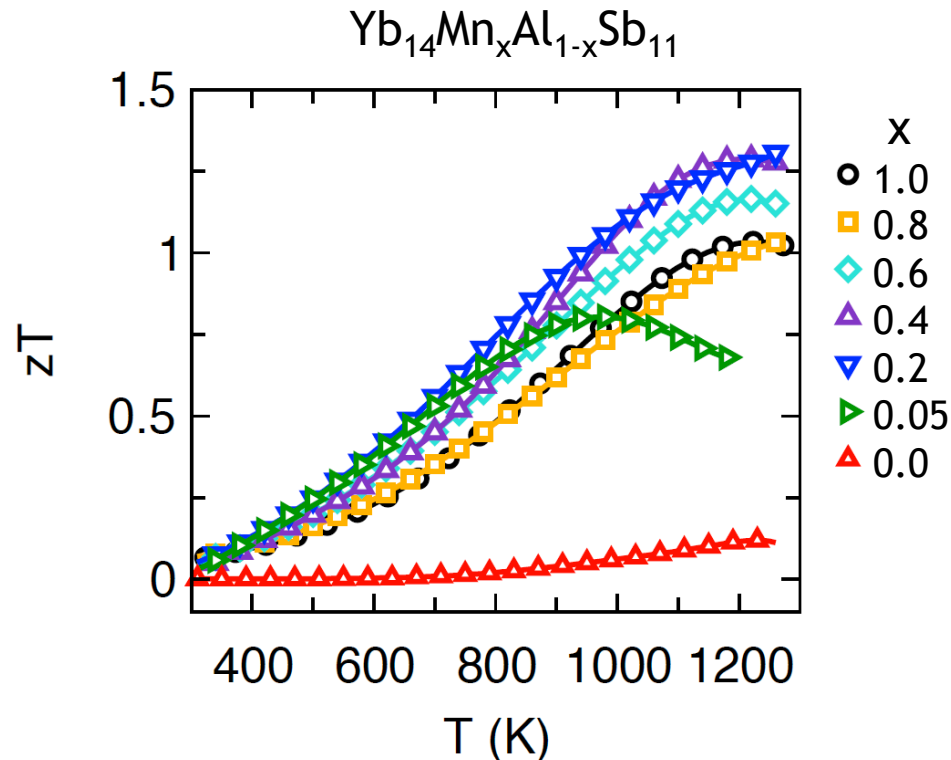


Figure of merit (zT) > 1 at 1200K!

NASA has been flying $\text{Si}_{0.8}\text{Ge}_{0.2}$ for the last 30 years with a peak zT of 0.6 at 1100K.

JPL:

14-1-11/ $\text{La}_{3-x}\text{Te}_4$ (BOL)

10% efficient at 1000°C

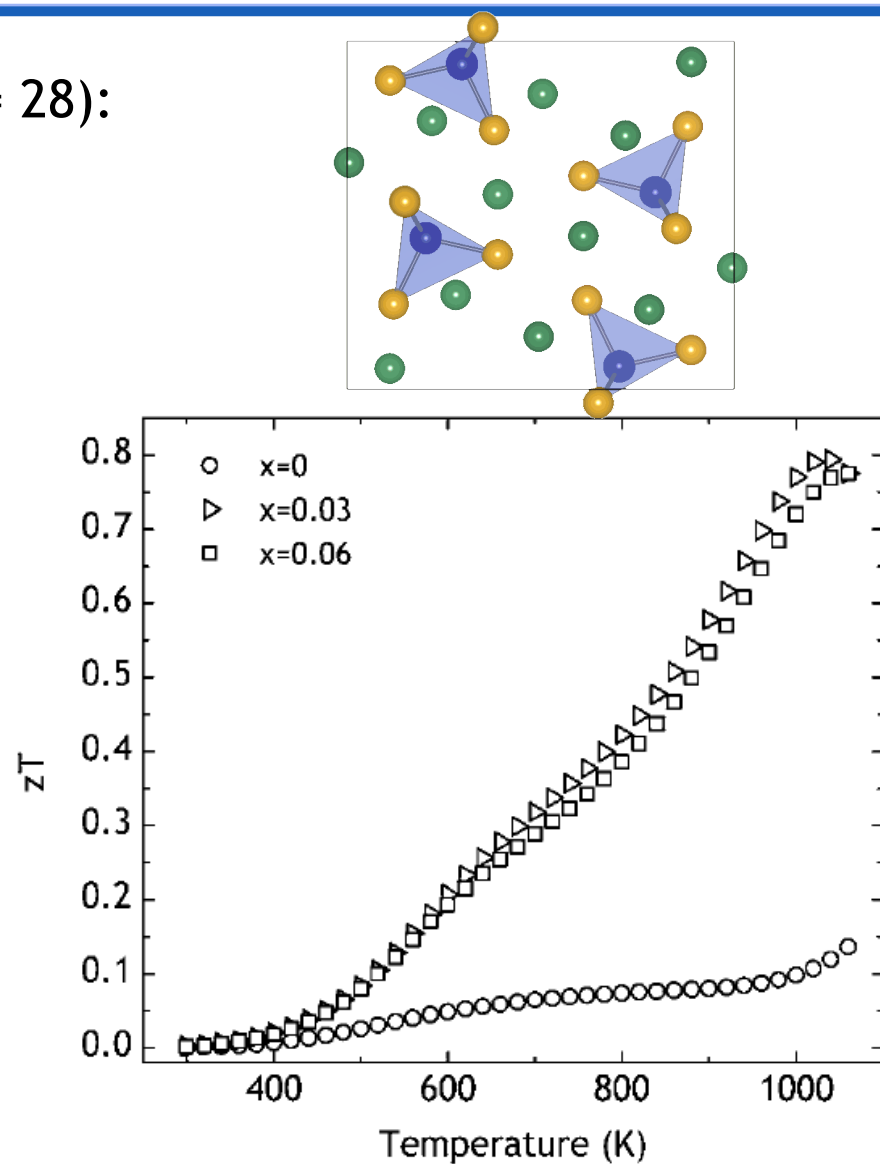
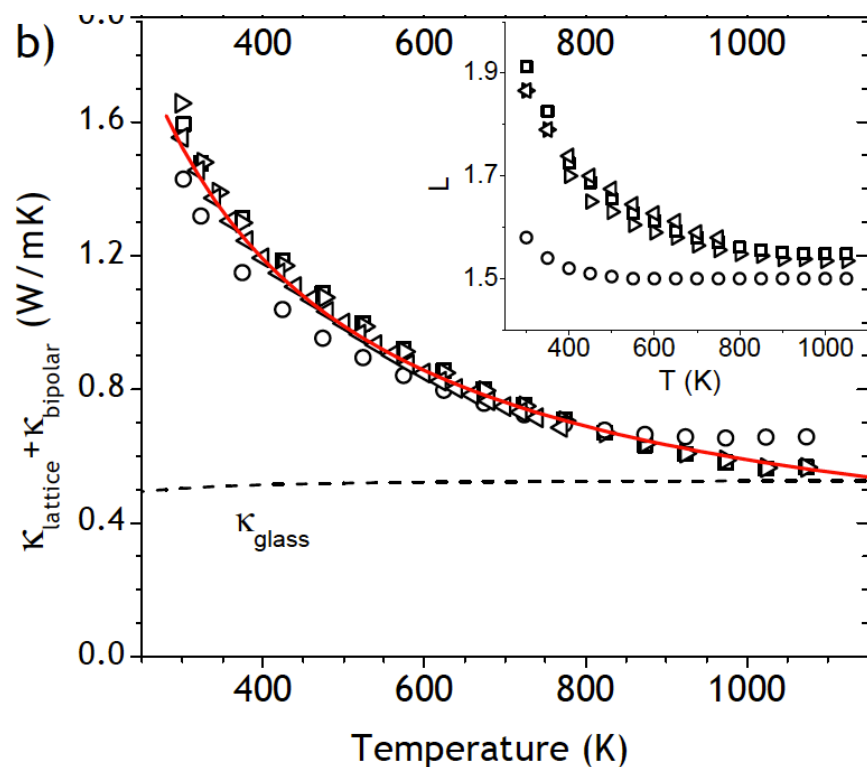
3000 hours of test

Segmented with skutterudite:

15% efficient (180/1000°C, BOL)

New Earth-abundant complex semiconductors

$\text{Ca}_5\text{Al}_2\text{Sb}_6$ (N = 26) and Ca_3AlSb_3 (N = 28):



Outline

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

Group velocity (v_g)

- Structural complexity
 - $\text{Yb}_{14}\text{MnSb}_{11}$
 - $\text{Ca}_5\text{Al}_2\text{Sb}_6$
 - Ca_3AlSb_3

Scattering (τ)

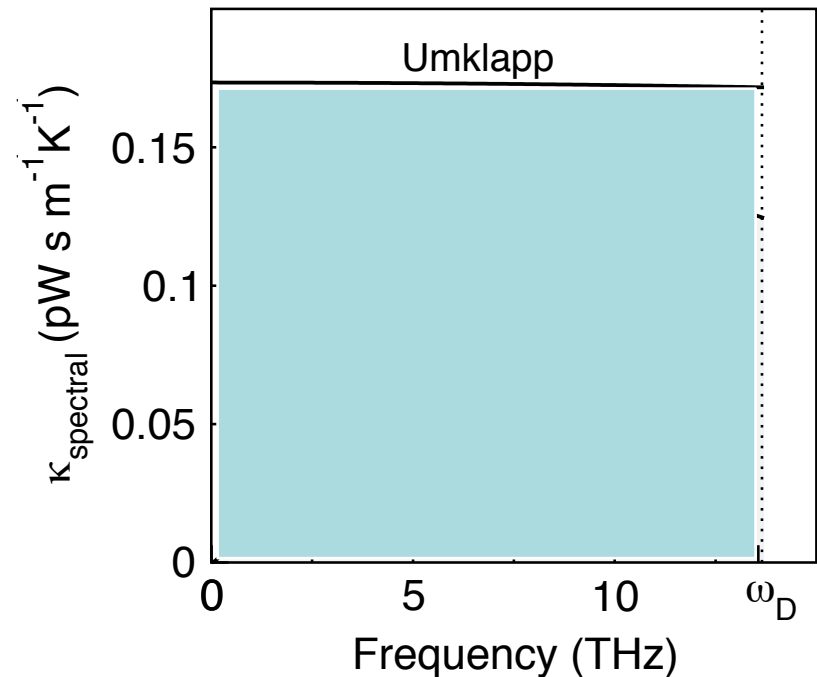
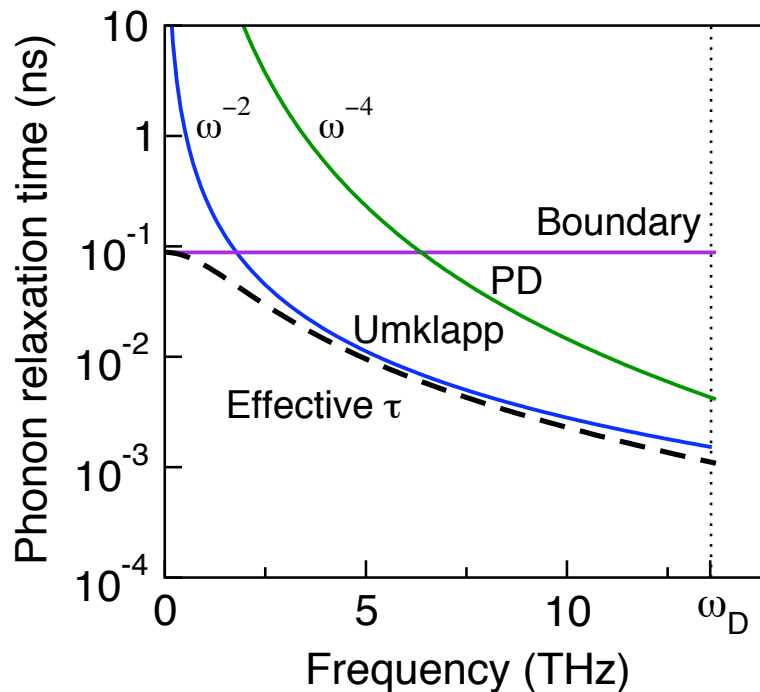
- Intrinsic point defects
 - $\text{La}_{3-x}\text{Te}_4$
 - Zn_4Sb_3
- Nanostructures
 - Zn_4Sb_3
 - PbTe

Scattering: Relevant phonon relaxation times

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

$$\tau^{-1} = \sum_i \tau_i^{-1}$$

$$\kappa_L = \int \kappa_{\text{spectral}} d\omega$$

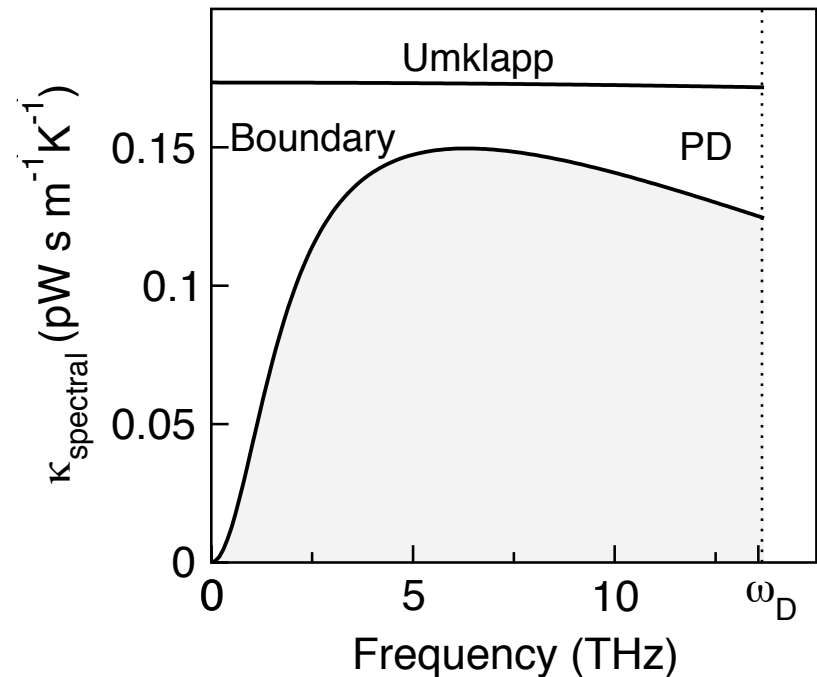
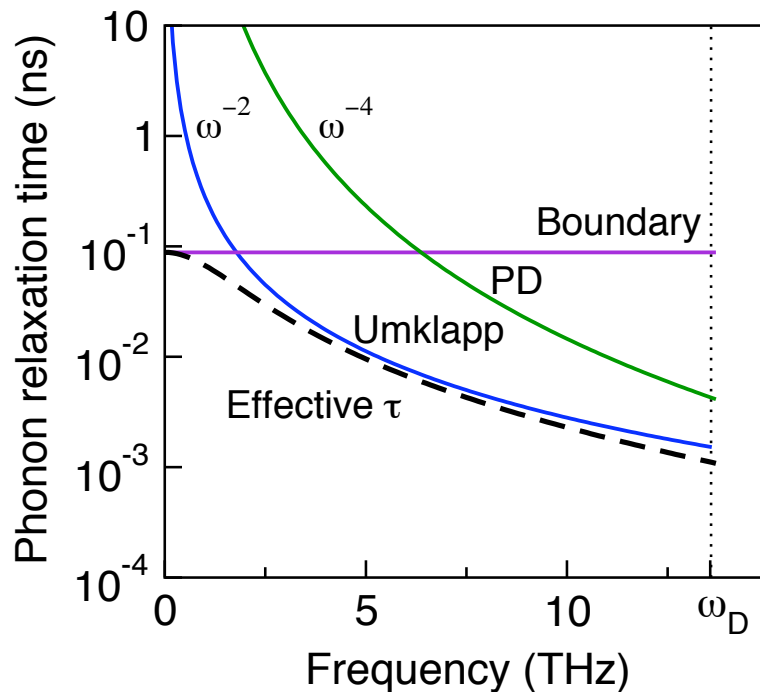


Scattering: Relevant phonon relaxation times

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

$$\tau^{-1} = \sum_i \tau_i^{-1}$$

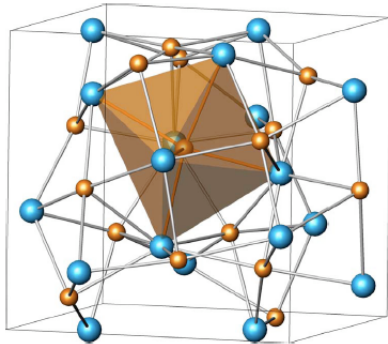
$$\kappa_L = \int \kappa_{\text{spectral}} d\omega$$



$\text{La}_{3-x}\text{Te}_4$ - High T *n*-type material

Charge balance: $\text{La}_2^{3+}\text{Te}_3^{2-}$

Th_3P_4 :



Conflict between structure type and charge balanced composition:

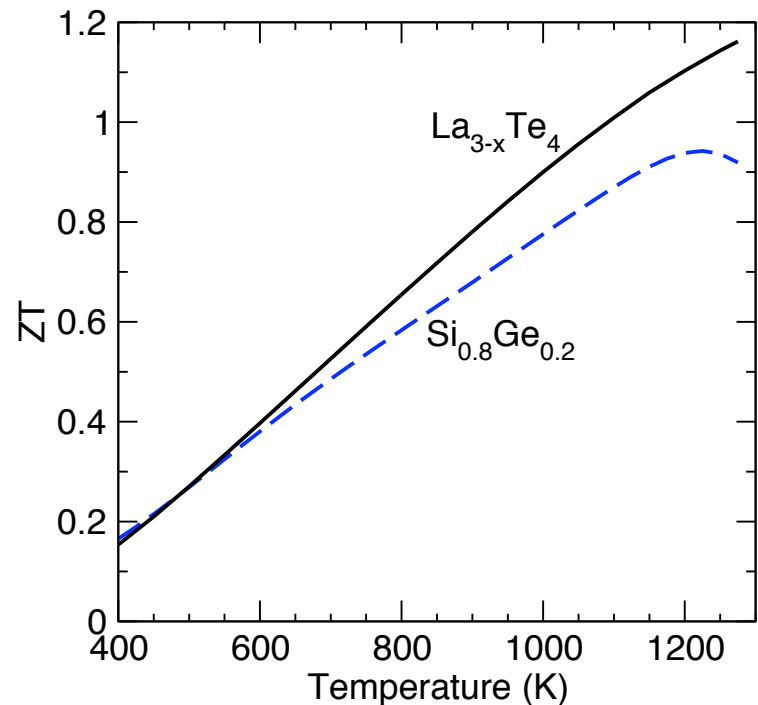
- **vacancies!**

Other sources of low κ_L :

- Anharmonicity (1.7)
- High density
- $N = 14$
- Moderately soft bonding
 $v = 3580$ (l) and 2010 (t) m/s

Cutler et al, Phys. Rev. (1964)

Wood, Rep. Prog. Phys. (1988)



May et al, Chem. Mater. (2010)

May et al, Phys. Rev. B (2010)

Delaire et al, Phys Rev B (2009)

May et al Phys. Rev. B (2009)

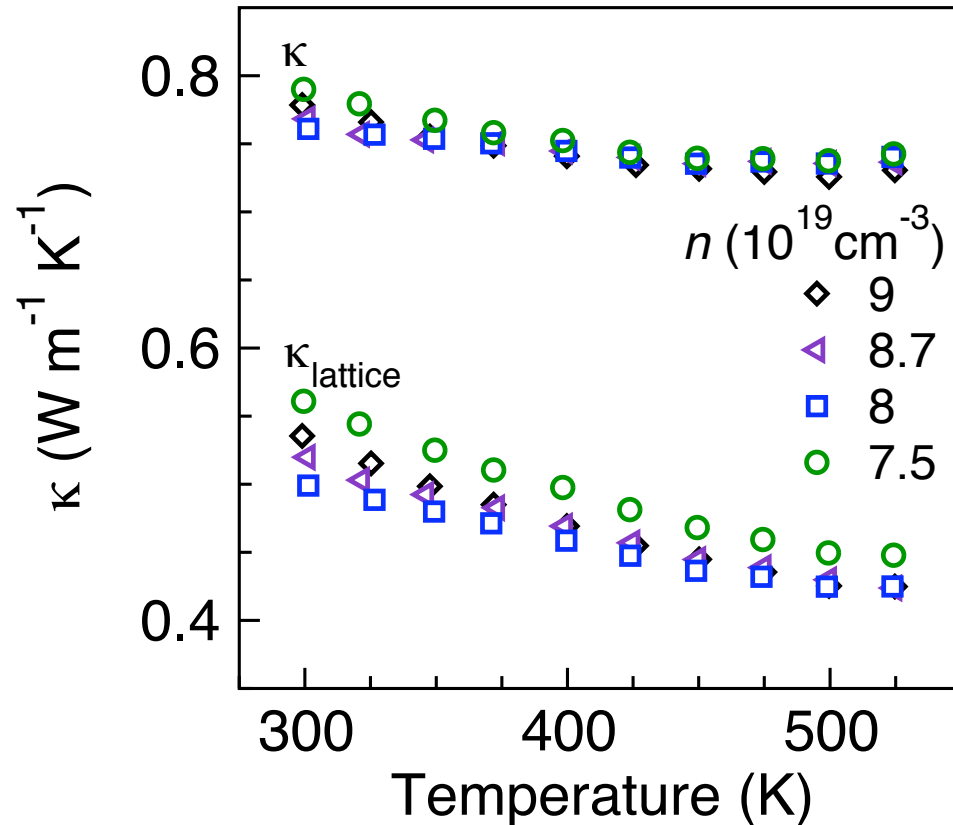
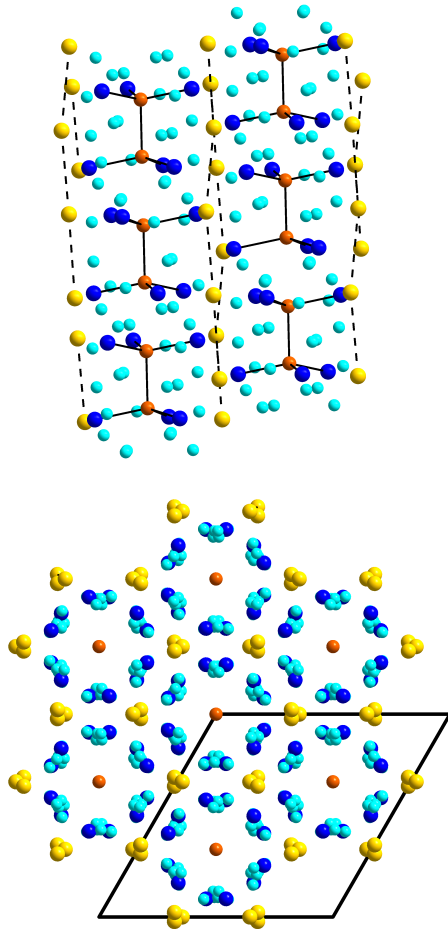
May et al Phys. Rev. B (2008)

Point defect scattering in Zn_4Sb_3

Like $\text{La}_{3-x}\text{Te}_4$, conflict between **structure** and **charge balance**

Structure: $\text{Zn}_{36}\text{Sb}_{30}$ Charge balance: $\text{Zn}_{39}\text{Sb}_{30}$

3 additional Zn - interstitials

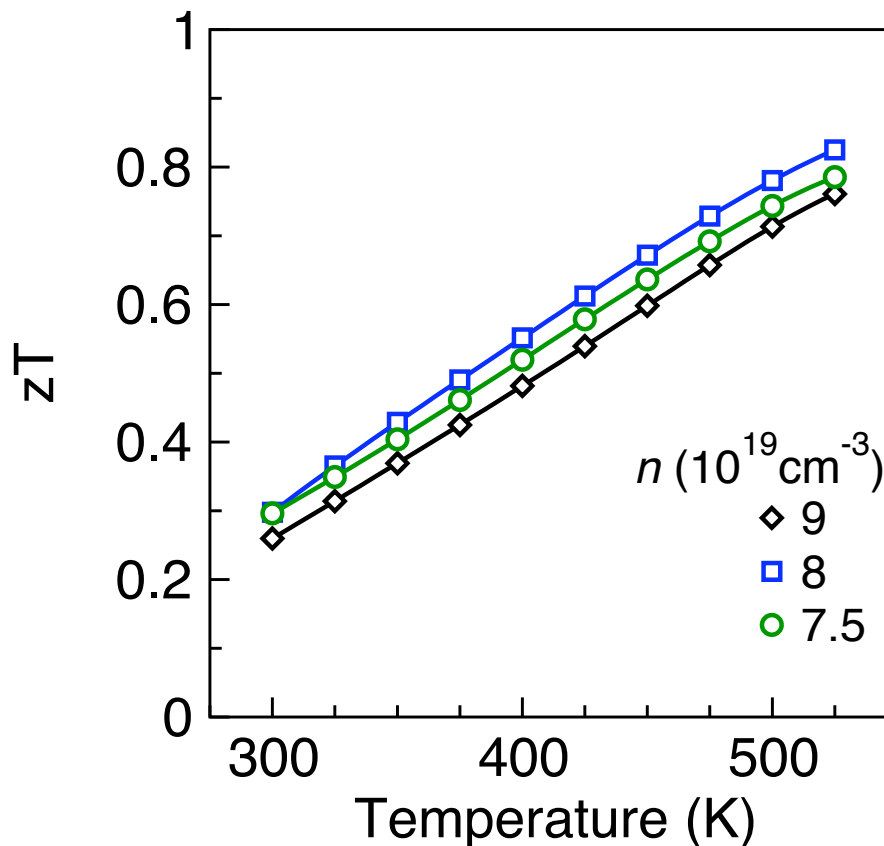
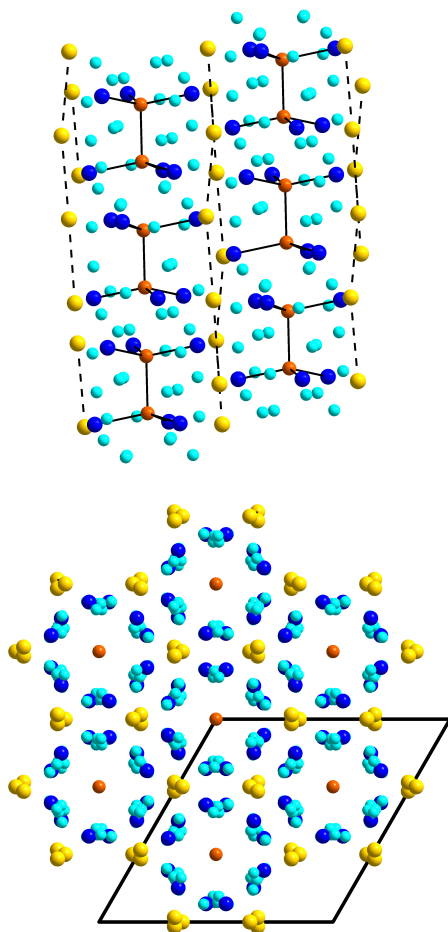


Zn_4Sb_3

Like $\text{La}_{3-x}\text{Te}_4$, conflict between **structure** and **charge balance**

Structure: $\text{Zn}_{36}\text{Sb}_{30}$ Charge balance: $\text{Zn}_{39}\text{Sb}_{30}$

3 additional Zn - interstitials



Outline

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

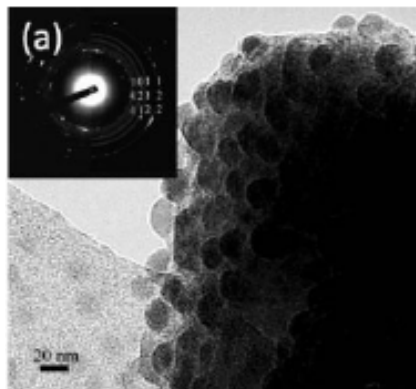
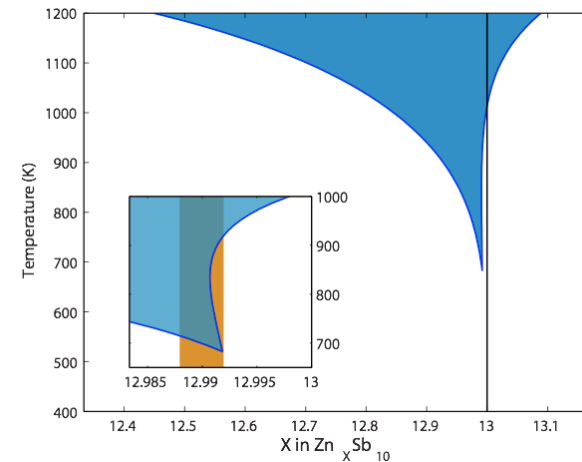
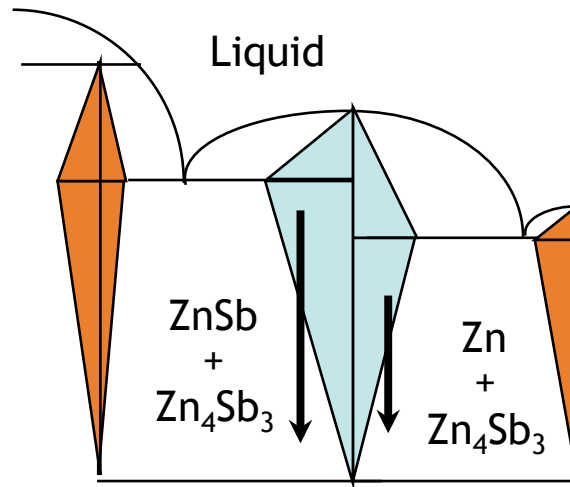
Group velocity (v_g)

- Structural complexity
 - $\text{Yb}_{14}\text{MnSb}_{11}$
 - $\text{Ca}_5\text{Al}_2\text{Sb}_6$
 - Ca_3AlSb_3

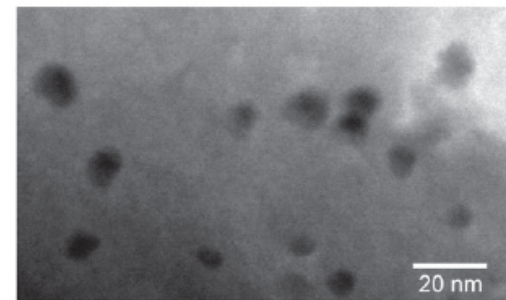
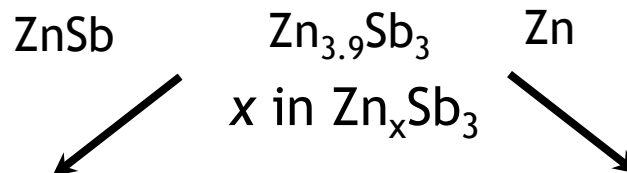
Scattering (τ)

- Intrinsic point defects
 - $\text{La}_{3-x}\text{Te}_4$
 - Zn_4Sb_3
- Nanostructures
 - Zn_4Sb_3
 - PbTe

Nanostructured Zn_4Sb_3



ZnSb precipitates (10nm)
found in Sb rich samples of
 Zn_4Sb_3

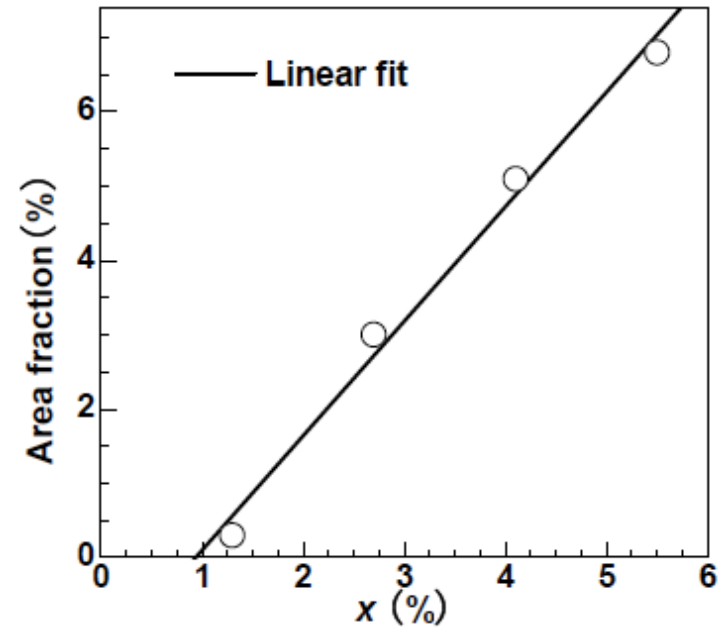
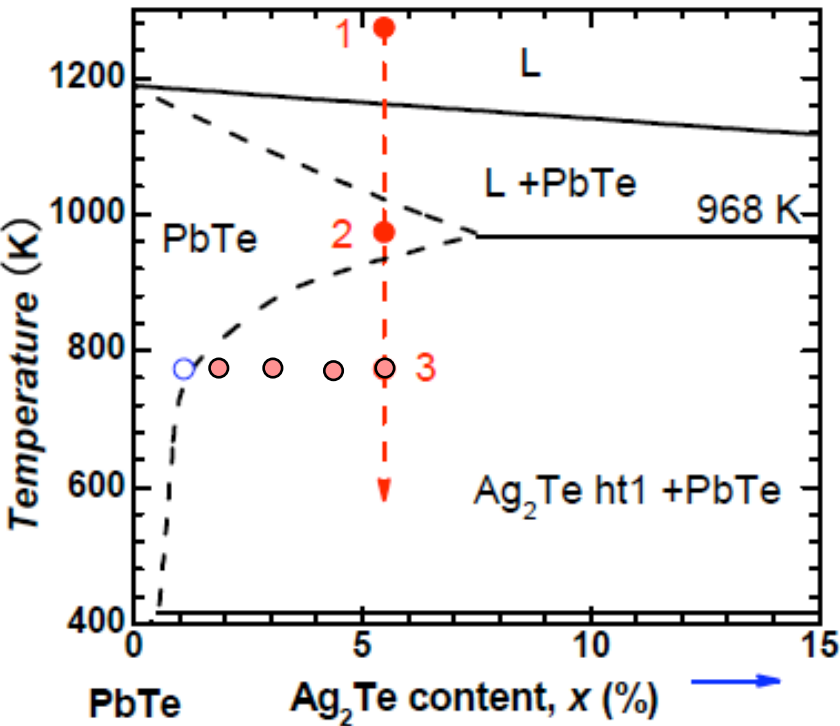


Coherent Zn precipitates
found in some samples of
 Zn_4Sb_3

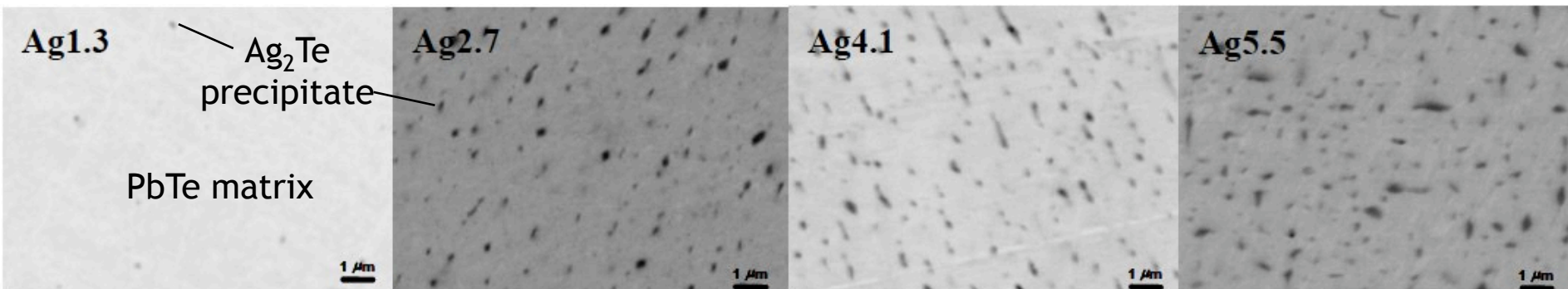
Prytz et al. Phil. Mag. Lett. (2009)
Toberer et al, J. Mater. Chem (2010)
Pohmrehn et al Phys. Rev B in press

PbTe-Ag₂Te Composite Formation

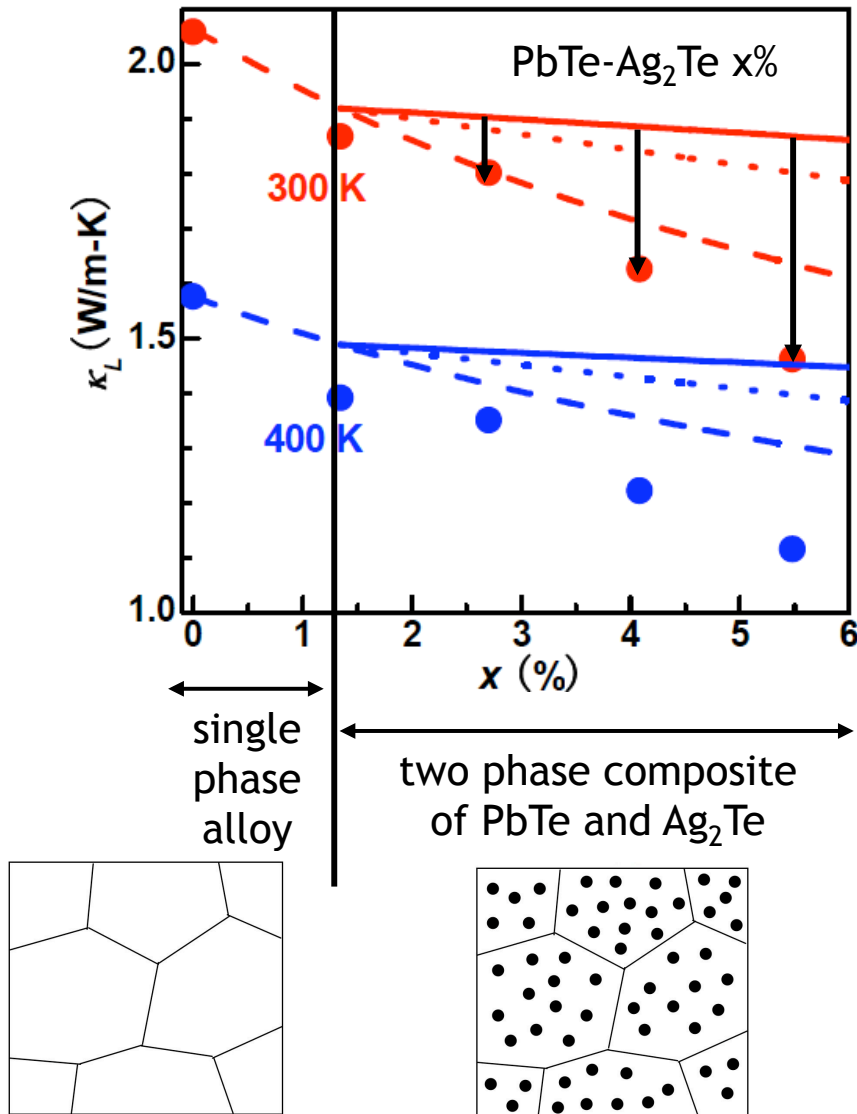
Ag₂Te precipitates form as expected from equilibrium phase diagram



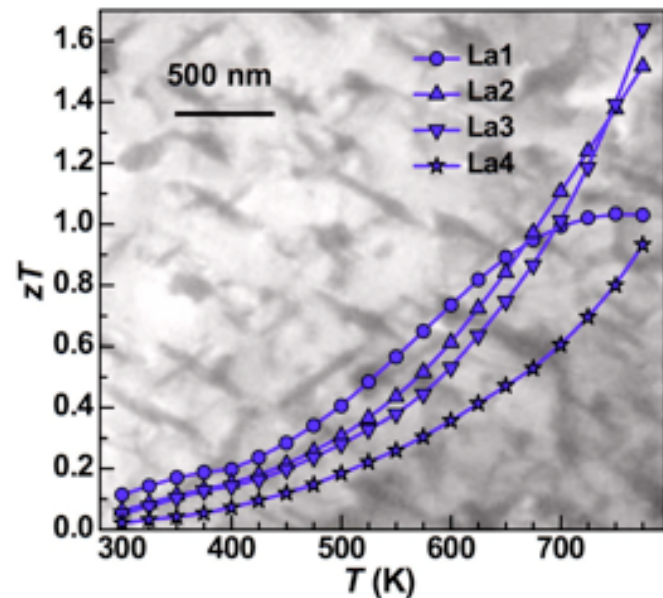
Pei, et al *Adv. Funct. Mat.*, (in Press)



Nanoscale Effects



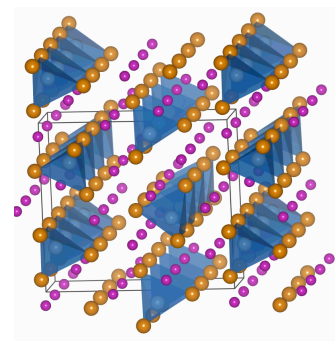
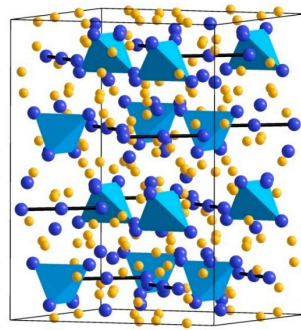
- Thermal conductivity reduction $\sim 20\%$
- Must be reduced in matrix
- Reduction due to Nanoscale Effects
- even from $\sim 100\text{nm}$ microstructure!



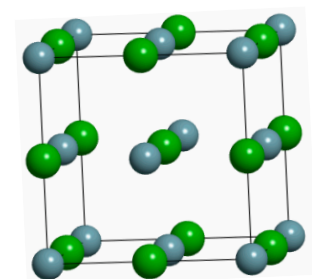
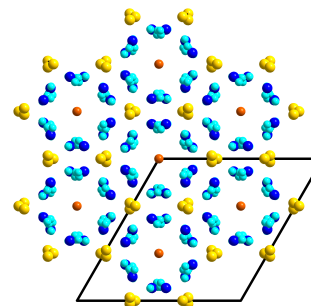
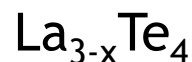
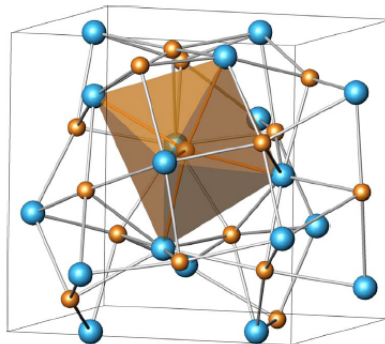
Outlook

Successful strategies for targeting low κ_L materials must utilize both **group velocity** and **scattering**!

Group velocity: heavy materials, soft bonding, low velocity optical modes



Scattering: spontaneously formation of point defects and nanostructures



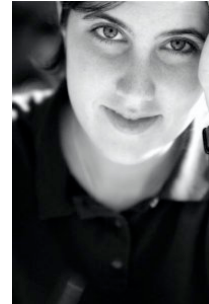
Acknowledgements



Jeff Snyder



Andrew May



Alex Zevalkink

Other groups for the research presented here:

Bo Iversen (U. Aarhus, Denmark)

Susan Kauzlarich (UC Davis)

Joseph Heremans (Ohio State U.)

Jean-Pierre Fleurial and Thierry Callait (Jet Propulsion Laboratory)



Review articles:

General: Snyder & Toberer, *Nature Mater.* 7 (2008) 105.

Zintl transport: Toberer, May, Snyder, *Chem. Mater.* 22 (2010) 624.

Lattice thermal conductivity model

Debye Model:

$$\kappa_L = \frac{1}{3} \int C(\omega) \nu_g(\omega)^2 \tau(\omega) d\omega \quad C_v = \frac{3\hbar^2}{2\pi^2 k_B T^2} \int_0^{\omega_m} \frac{\omega^4 \exp[\hbar\omega/kT]}{\nu_p^2 \nu_g (\exp[\hbar\omega/kT] - 1)^2} d\omega$$
$$\kappa_L = \frac{\hbar^2}{2\pi^2 k_B T^2} \int_0^{\omega_m} \frac{\omega^4 \exp[\hbar\omega/kT]}{\nu_p^2 (\exp[\hbar\omega/kT] - 1)^2} \nu_g \tau d\omega$$

Scattering:

$$\tau^{-1} = \sum_i \tau_i^{-1} \quad \tau_U(\omega) = \frac{\bar{M} \nu_g \nu_p^2}{C a \gamma^2 k \omega^2} \quad \tau_B = \frac{d}{\nu}$$
$$\tau_{PD} = \frac{V}{4\pi \nu^3} \omega^4 \sum_i f_i \left(\frac{\bar{m} - m_i}{\bar{m}} \right)^2$$

Cahill's glass model:

$$\kappa_g = \left(\frac{\pi}{6} \right)^{1/3} \frac{\hbar^4}{k_B^3 a^2 T^2} \sum_i \frac{\nu_i}{\Theta_i^2} \int_{\frac{\omega_D}{n^{1/3}}}^{\omega_D} \frac{\omega^3 \exp[\hbar\omega/kT]}{(\exp[\hbar\omega/kT] - 1)^2} d\omega$$

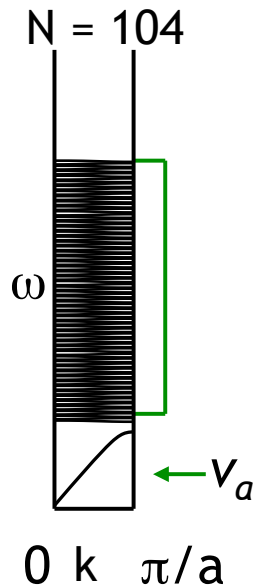
Understanding thermal conductivity in Yb₁₄MnSb₁₁

Phonon mean free path (l):

$$l^{-1} = \sum_i l_i^{-1} \quad l_{Umklapp}(\omega) = \frac{\overline{M} v_g^2 v_p^2}{C a \gamma^2 k \omega^2}$$

$$l_{Boundary}(\omega) = d$$

$d = 1 \text{ micron}$



$$\kappa_{acoustic} = \frac{1}{3} \int_0^{\omega_{acoustic}} C v l d\omega$$

300 K

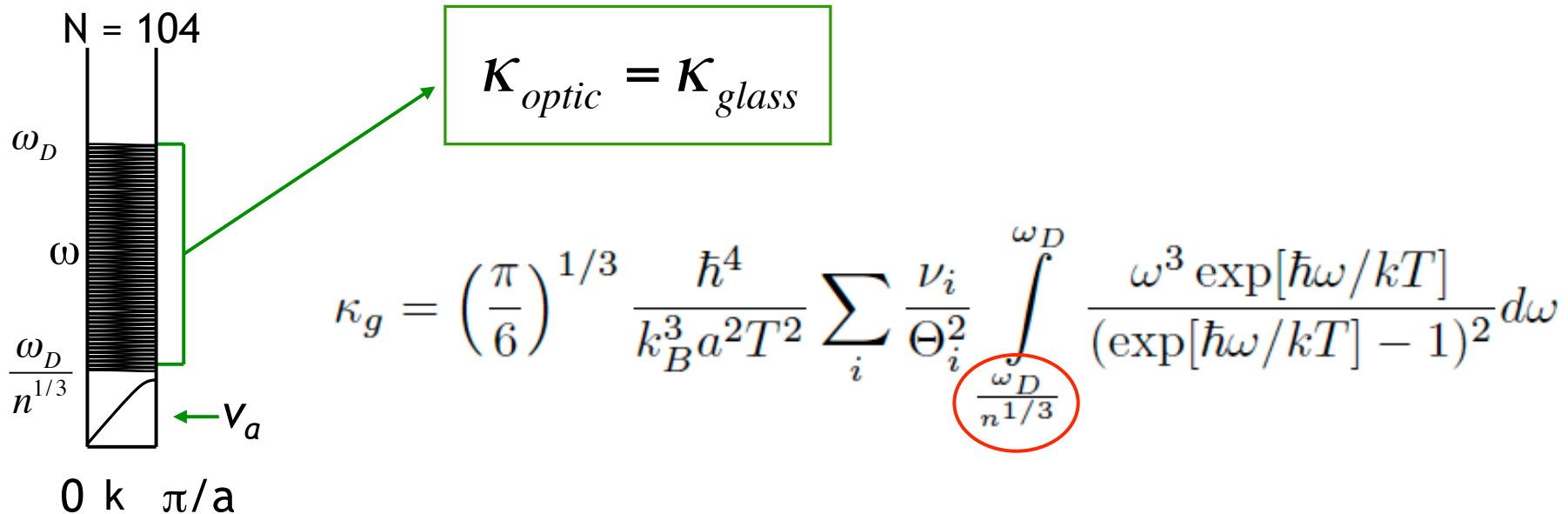
$0.5 \text{ W m}^{-1} \text{ K}^{-1}$

300K

Prediction acoustic: $0.5 \text{ W m}^{-1} \text{ K}^{-1}$

Reality total: $0.6 \text{ W m}^{-1} \text{ K}^{-1}$

Thermal conductivity in $\text{Yb}_{14}\text{MnSb}_{11}$



Here, heat transport is treated as a random walk of energy on a lattice. Coupling constant between the harmonic oscillators estimated by v .

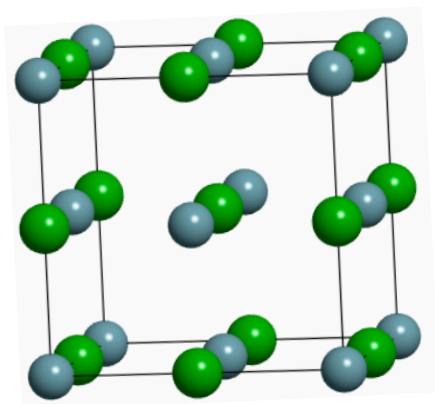
Speed of sound (v)

Despite similar speed of sounds between PbTe and Yb₁₄MnSb₁₁....
..... very different lattice thermal conductivities.

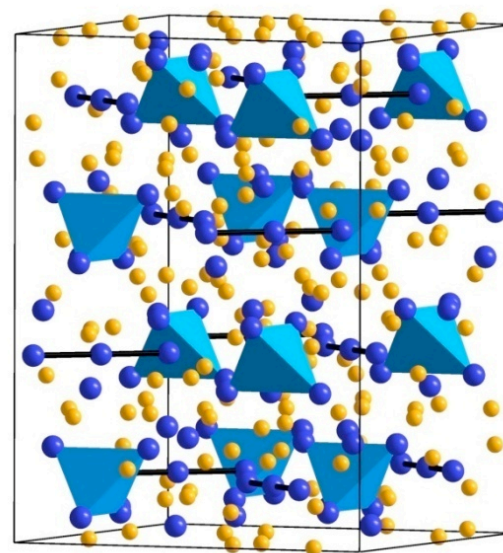
$$v = \sqrt{\frac{B}{d}}$$



	d g cm ⁻³	v_M m/s	κ_L (300 K) W m ⁻¹ K ⁻¹
PbTe	8.2	1400	2.1
Yb ₁₄ MnSb ₁₁	8.4	1600	0.6



PbTe (Fm-3m)



Yb₁₄MnSb₁₁ (I4/ammc)

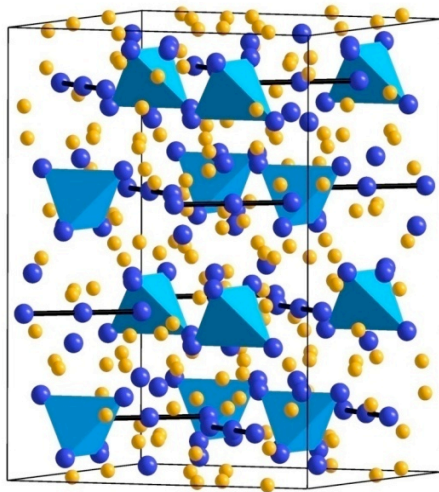
$\text{Yb}_{14}\text{AlSb}_{11}$

Cations:

14 Yb^{2+}

Isolated anionic moieties:

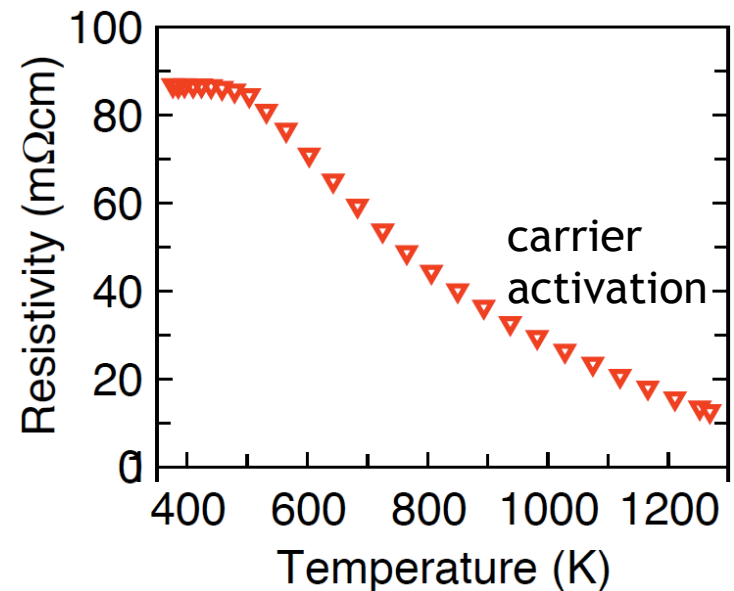
AlSb_4^{9-} tetrahedra
 Sb_3^{7-} linear trimer
4 isolated Sb^{3-}



104 atoms in the primitive cell.

Intrinsic semiconductor due to precise valence balance

- Large, decreasing resistance with increasing T



Rauscher, Toberer, et al, Chem. Mater (2010)

Cox, Toberer, et al, Chem. Mater. (2009)

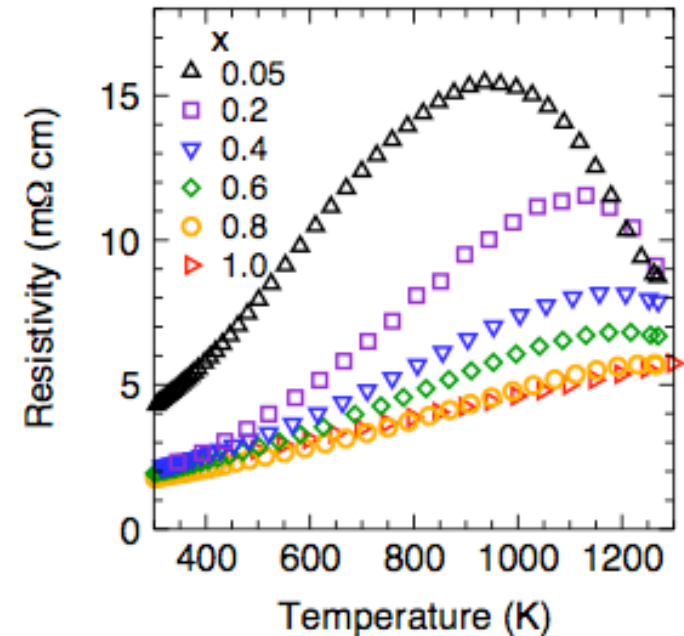
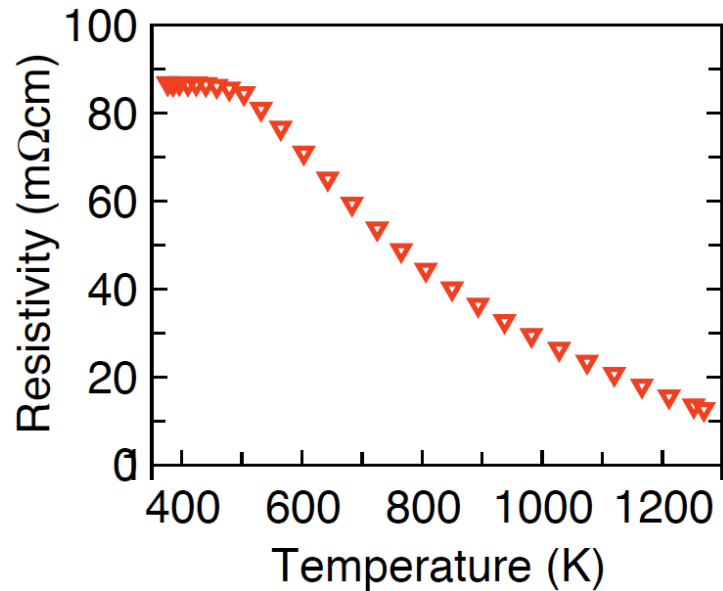
Toberer et al Appl. Phys. Lett. (2008)

Brown, Toberer, et al, Chem. Mater. (2008)

→ Toberer et al, Adv. Funct. Mater. (2008)

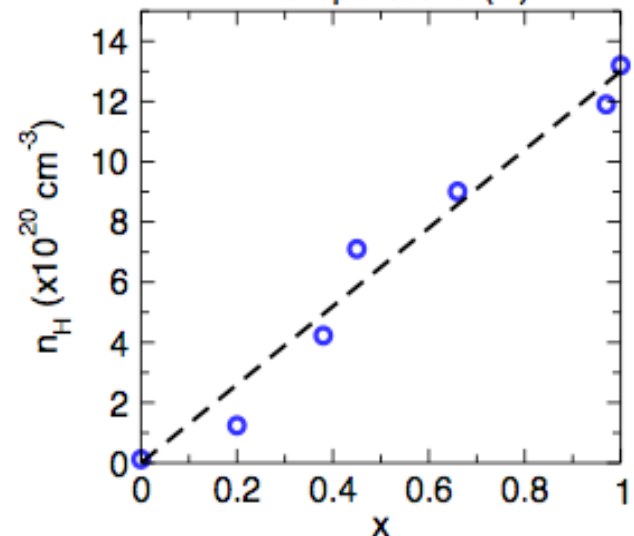
Brown et al Chem. Mater. (2006)

$\text{Yb}_{14}\text{Mn}_x\text{Al}_{1-x}\text{Sb}_{11}$ - Metallic resistivity!

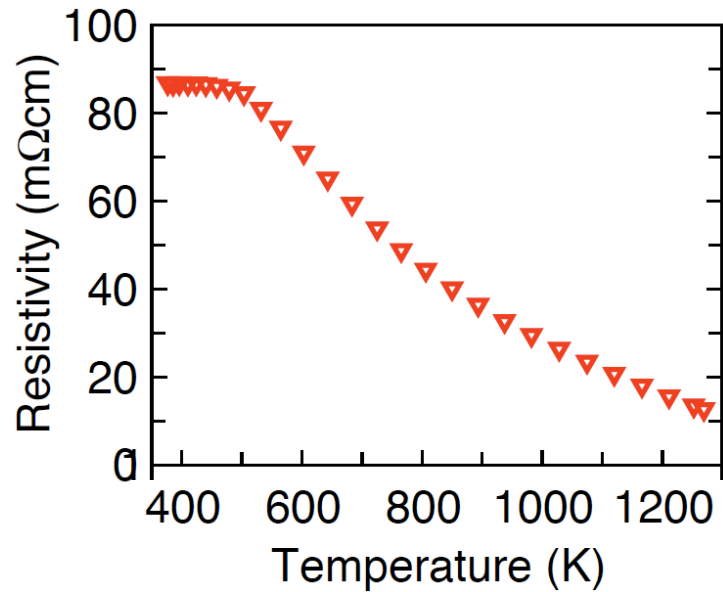


Successful p -type doping
with Mn^{2+} alloying on Al^{3+} site!

$\text{Yb}_{14}\text{MnSb}_{11}$ very metallic
(linear resistivity with temperature)



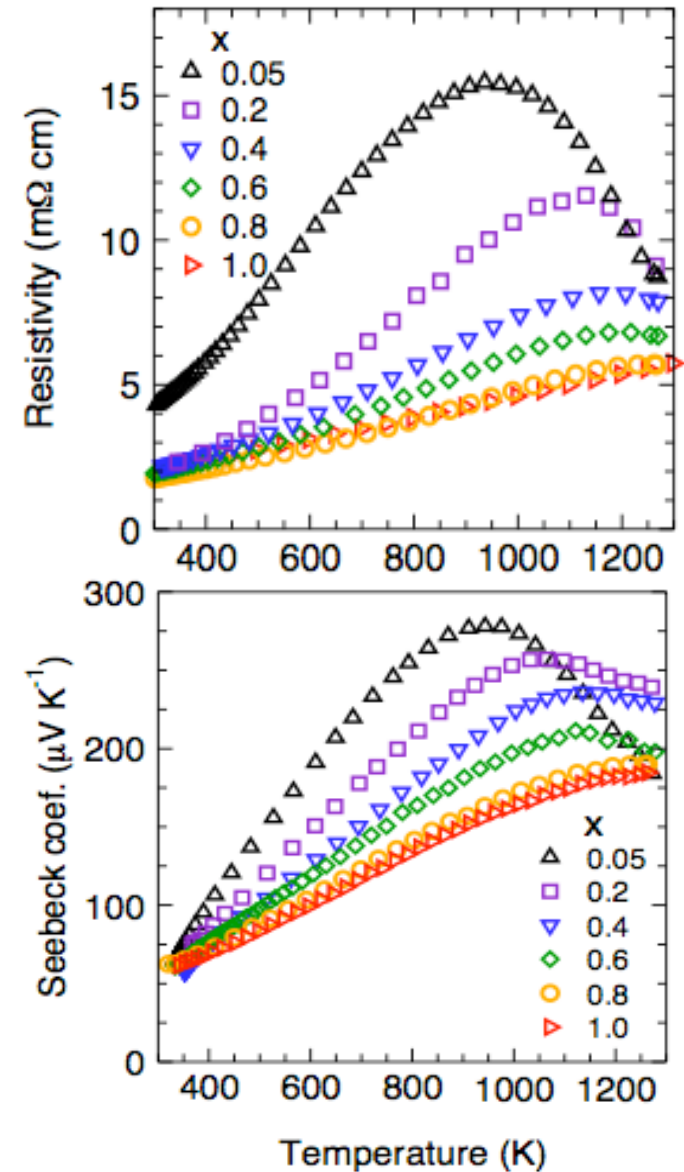
$\text{Yb}_{14}\text{Mn}_x\text{Al}_{1-x}\text{Sb}_{11}$ - Metallic resistivity!



$$\alpha = \frac{8\pi^2 k_B^2}{3eh^2} m^* T \left(\frac{\pi}{3n} \right)^{2/3}$$

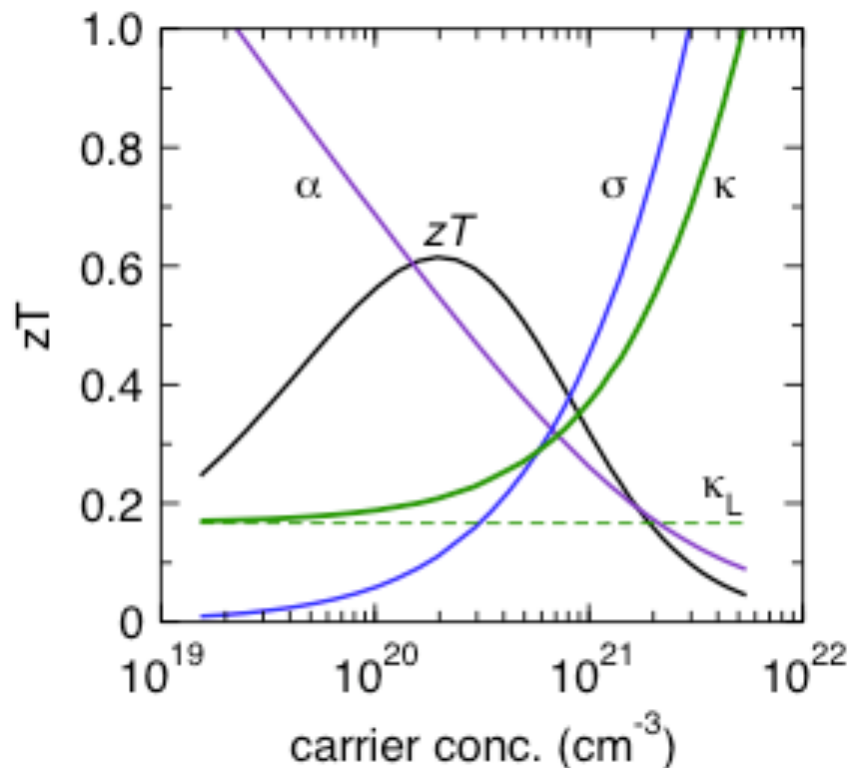
$$m^* = 3 m_e$$

Massive for a band conductor!



Beyond the confines of a single parabolic band model

Transport within a single parabolic band will only get us so far - need a new approach!



$$\alpha = \frac{k}{e} \left(\frac{(2 + \lambda)F_{1+\lambda}(\eta)}{(1 + \lambda)F_{\lambda}(\eta)} - \eta \right)$$

$$F_r(\eta) = \int_0^{\infty} \zeta^r f_0(\eta) d\zeta$$

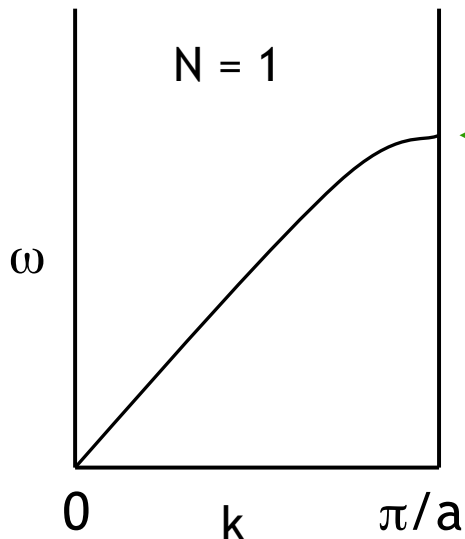
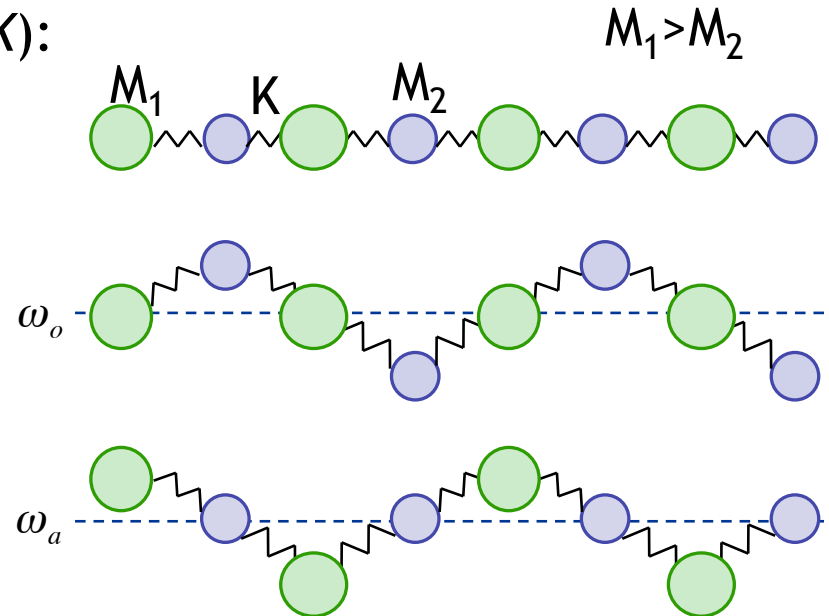
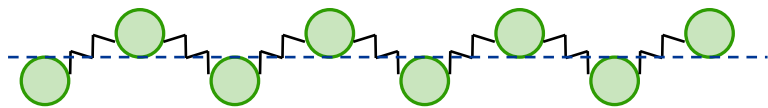
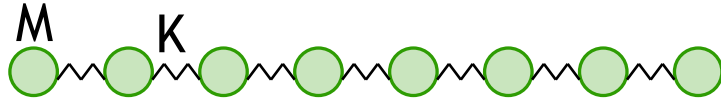
$$\mu_H = \frac{\mu_0 \pi^{1/2} F_{\lambda}(\eta)}{2\Gamma(1 + \lambda) F_{1/2}(\eta)}$$

$$n = \frac{4}{\sqrt{\pi}} \left(\frac{2\pi m^* kT}{h^2} \right)^{3/2} F_{1/2}(\eta)$$

$$L = \left(\frac{k}{e} \right)^2 \frac{3F_0(\eta)F_2(\eta) - 4F_1(\eta)^2}{F_0(\eta)^2}$$

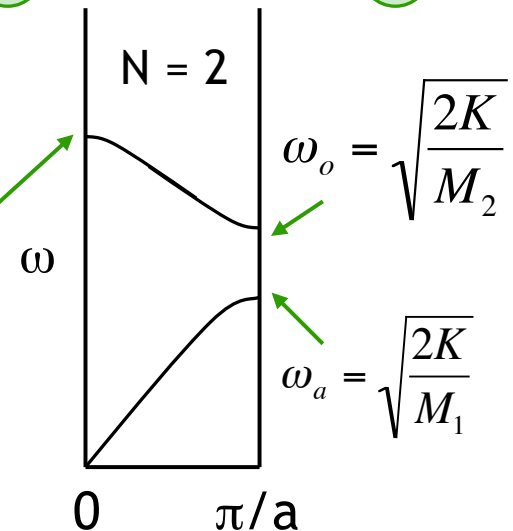
Formation of band gaps in the phonon dispersion

1D chain of atoms coupled via spring (K):



$$\omega_{\max} = \sqrt{\frac{4K}{M}}$$

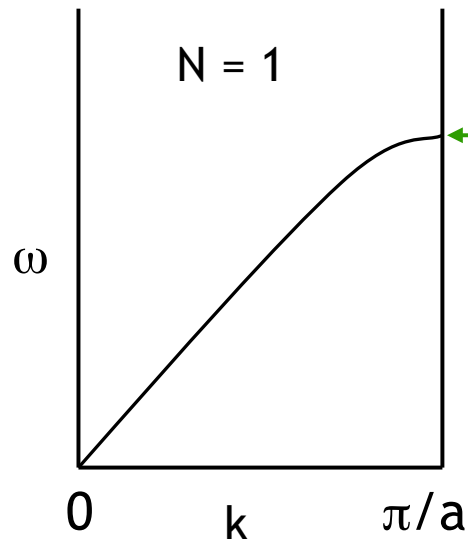
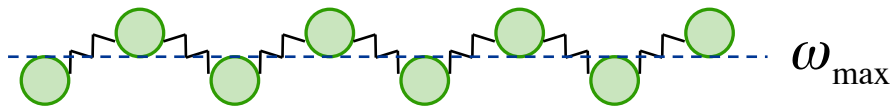
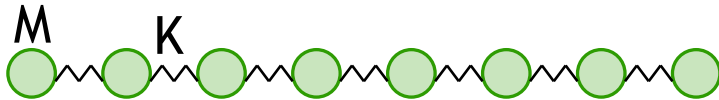
$$\omega_{\max} = \sqrt{\frac{2K}{M_1} + \frac{2K}{M_2}}$$



Phonon dispersion for 1-D chain

This approach to

1D chain of atoms coupled via spring (K):



$$\omega_{\max} = \sqrt{\frac{4K}{M}}$$

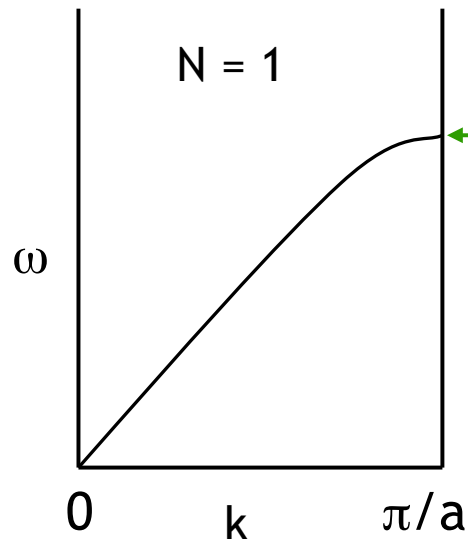
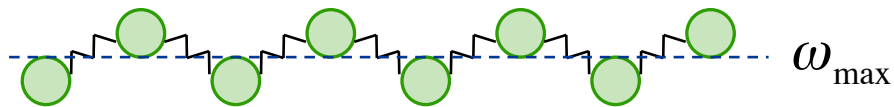
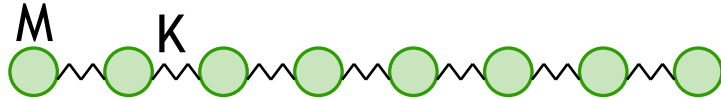
$$\kappa_{lattice} = \frac{1}{3} C_v v l$$

Fine approximation, if v is constant

$$v_g = \frac{\partial \omega}{\partial k}$$

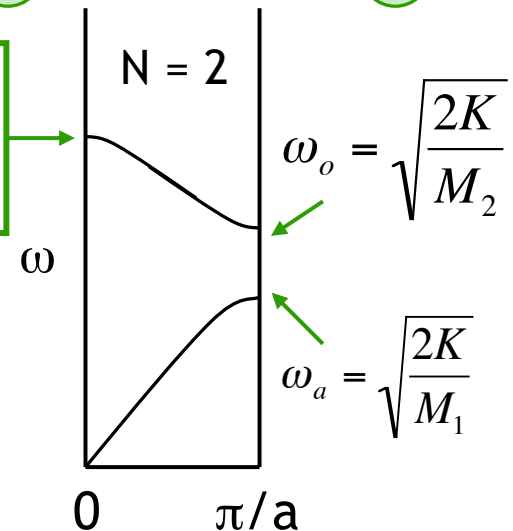
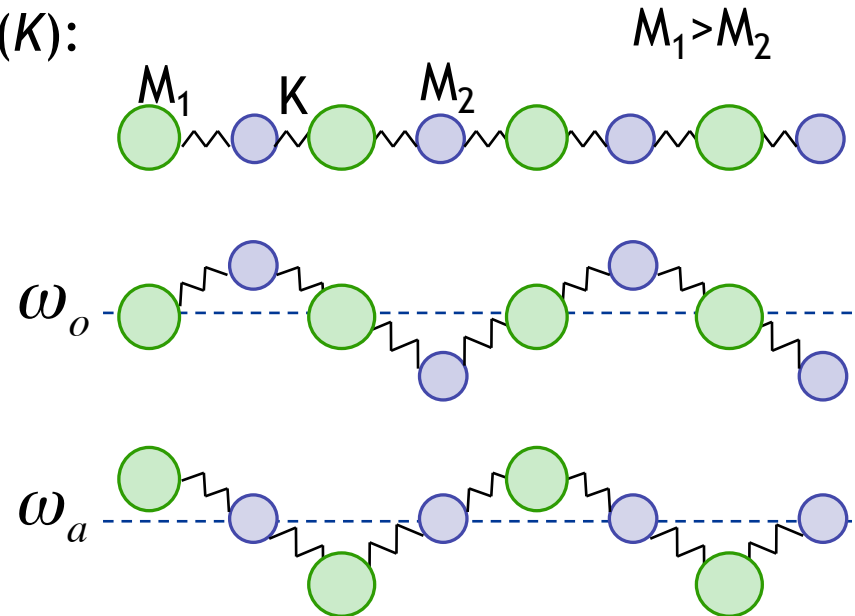
Formation of band gaps in the phonon dispersion

1D chain of atoms coupled via spring (K):



$$\omega_{\max} = \sqrt{\frac{4K}{M}}$$

$$\omega_{\max} = \sqrt{\frac{2K}{M_1} + \frac{2K}{M_2}}$$



Thermal conductivity estimation

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

v_g - phonon group velocity
 τ - phonon relaxation time
Umklapp, point defect...

Assumptions:

- 1- Acoustic branch: Debye solid, optical branches: phonon glass
- 2 - τ dominated by Umklapp scattering

$$\tau_{Umklapp} \propto T^{-1} \gamma^{-2}$$

Thermal conductivity estimation

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

v_g - phonon group velocity
 τ - phonon relaxation time
Umklapp, point defect...

Assumptions:

- 1- Acoustic branch: Debye solid, optical branches: phonon glass
- 2 - τ dominated by Umklapp scattering

$$\kappa_L = \frac{c \bar{M} \theta_D^3 a}{N^{1/3} \gamma^2 T}$$

$$\tau_{Umklapp} \propto T^{-1} \gamma^{-2}$$

M - mass
 θ_D - Debye temperature
 γ - Gruneisen parameter
 a - interatomic distance
 N - number of atoms in primitive cell

Good thermoelectric materials:

- heavy (low θ_D)
- anharmonic bonding (large γ)
- structurally complex (large N)

Recall $\theta_D \propto M^{-1}$

Search for improved materials

Want to satisfy 2 requirements:

- 1 - Degenerate semiconductor w/ large $m^{*3/2}\mu$, high degeneracy
- 2 - Structural complexity to trap heat in low ν optical modes

But where?

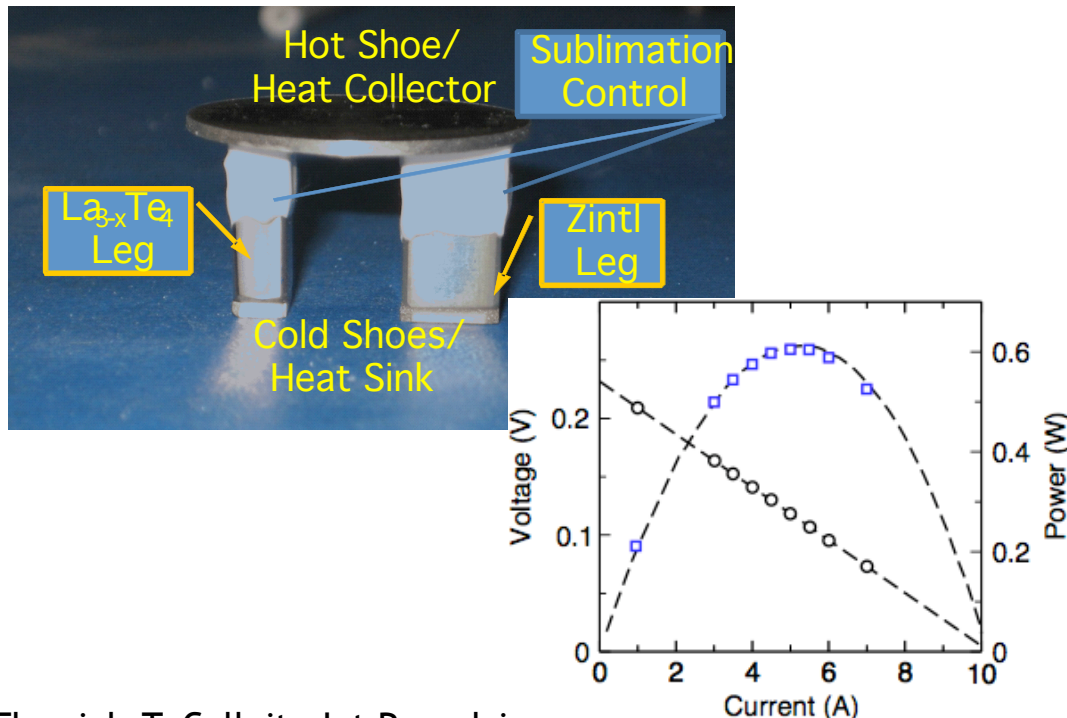
Unsegmented & Segmented Couples Under Development at JPL (10-15% eff.):

Uses novel materials developed in last few years:

- JPL and university collaborators
- 14-1-11 Zintl, $\text{La}_{3-x}\text{Te}_4$, nano bulk Si-Ge, filled Skutterudites

First set of couples built and tested at JPL in March 2009

- 10% efficient at 1000 C 14-1-11 Zintl/ $\text{La}_{3-x}\text{Te}_4$ (BOL), 3000 hours of test



*~ 6.3% Efficiency, 2.8 W/kg
Multi-Mission RTG (MSL)*

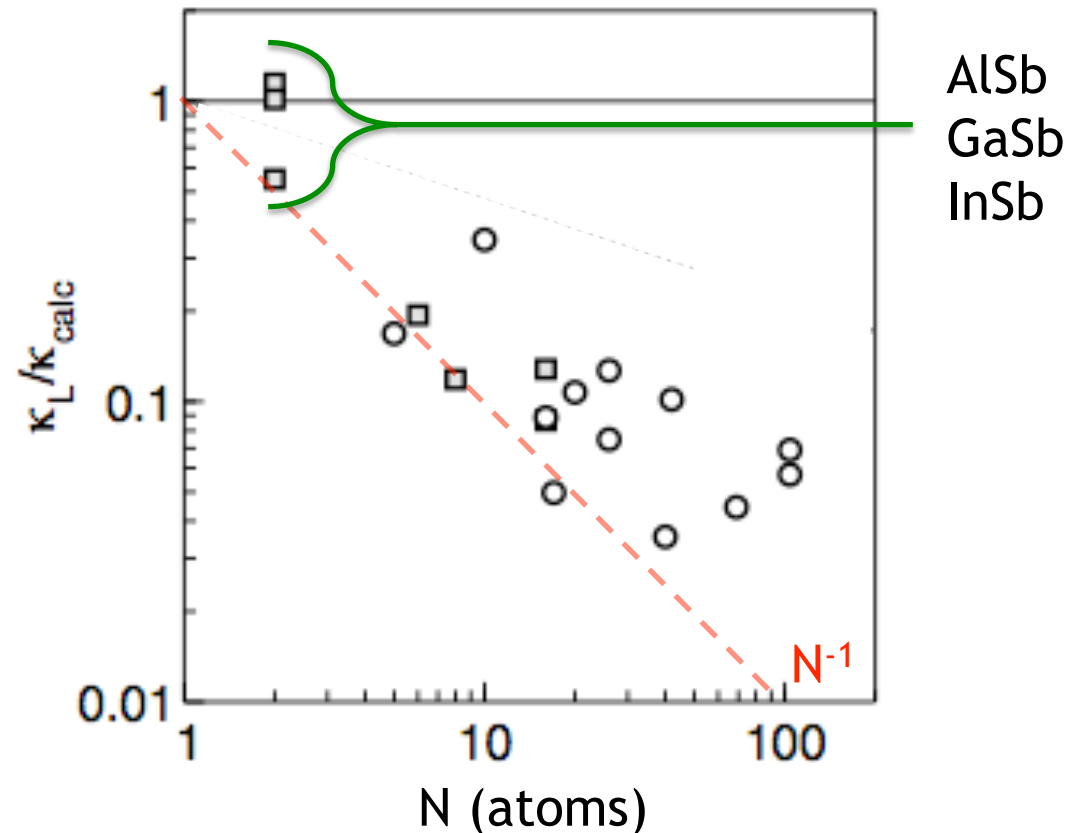
Near Term Goal:
> 10% Efficiency, 6 W/kg

Longer Term Goal:
> 15% Efficiency, 10 W/kg

Validating strong N dependence in antimonides

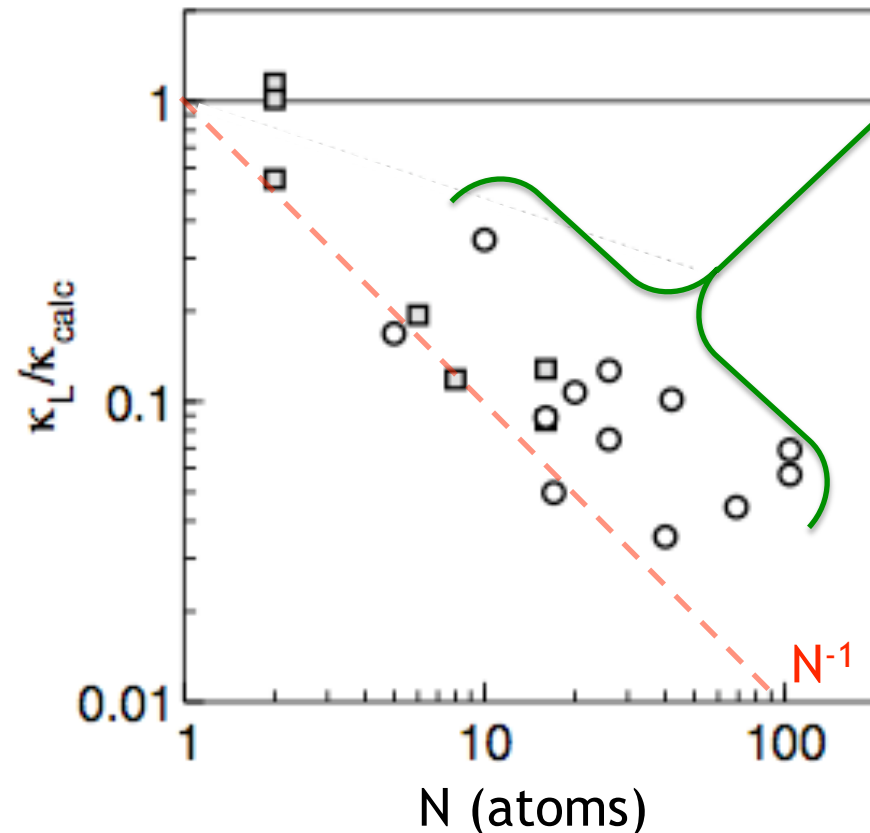
Strip N dependence from equation, compare to experiment:

$$\frac{\kappa_{L,\text{exp}}}{\kappa_{L,\text{calc}}} = \frac{\kappa_L \gamma^2 T}{c \bar{M} \theta_D^3 a}$$



Validating strong N dependence in antimonides

$$\frac{\kappa_{L,\text{exp}}}{\kappa_{L,\text{calc}}} = \frac{\kappa_L \gamma^2 T}{c \bar{M} \theta_D^3 a}$$



IrSb₃
 LiZnSb
 Mo₃Sb₇
 SrZn₂Sb₂
 CeFe₄Sb₁₂
 BaZn₂Sb₂
 Ca₅Al₂Sb₆
 Mg₃Sb₂
 SrZnSb₂
 Yb₁₁Sb₁₀
 ZnSb
 CdSb
 Yb₁₁GaSb₉
 Zn₄Sb₃
 Yb₁₄MnSb₁₁
 Yb₁₄AlSb₁₁

N dependence is strong!

Unsegmented & Segmented Couples Under Development at JPL (10-15% eff.):

Uses novel materials developed in last few years:

- JPL and university collaborators
- 14-1-11 Zintl, $\text{La}_{3-x}\text{Te}_4$, nano bulk Si-Ge, filled Skutterudites

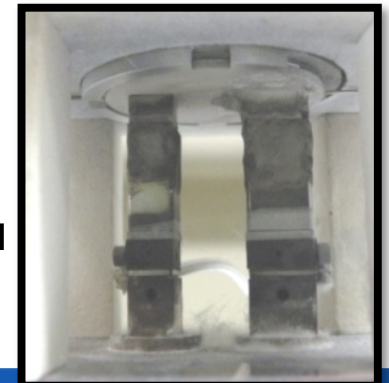
First set of couples built and tested at JPL in March 2009

- 10% efficient at 1000 C 14-1-11 Zintl/ $\text{La}_{3-x}\text{Te}_4$ (BOL), 3000 hours of test

Segmented couples recently built, tested

- **14-1-11, $\text{La}_{3-x}\text{Te}_4$** , segmented with filled **Skutterudites**
- >11% efficient at 800 C/200 C (BOL demonstration completed)
- >13% efficient at 1000 C/200 C (BOL demonstration completed)
- >15% efficient at 1000 C/200 C demonstration planned for 9/2011

**Advanced
Segmented
Couples**



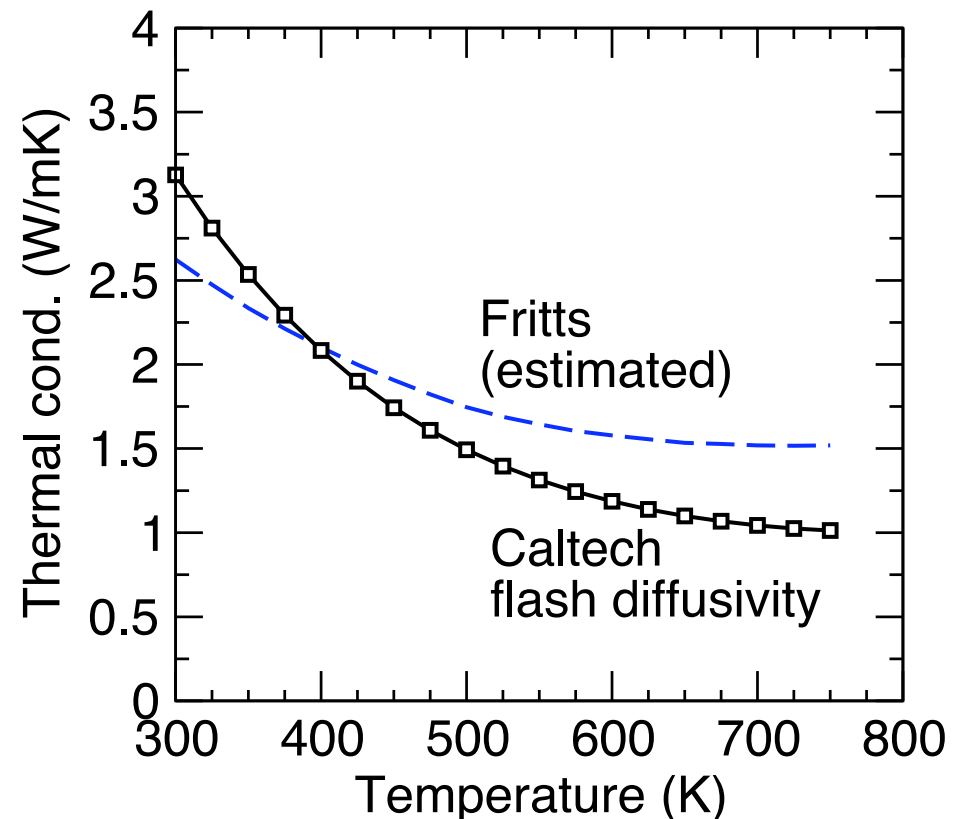
Reassessment of PbTe thermal conductivity

Classic 1960s work prior to development of laser flash diffusivity
High temperature κ was estimated!

Sources of low κ_L :

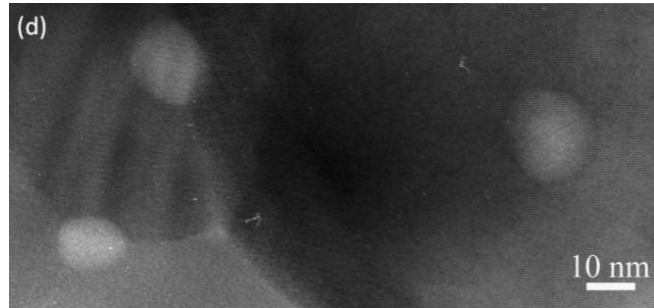
- Large Gruneisen parameter (1.5)
- High density
- Soft bonding

$v = \text{XXXX (l) and YYYy (t) m/s}$



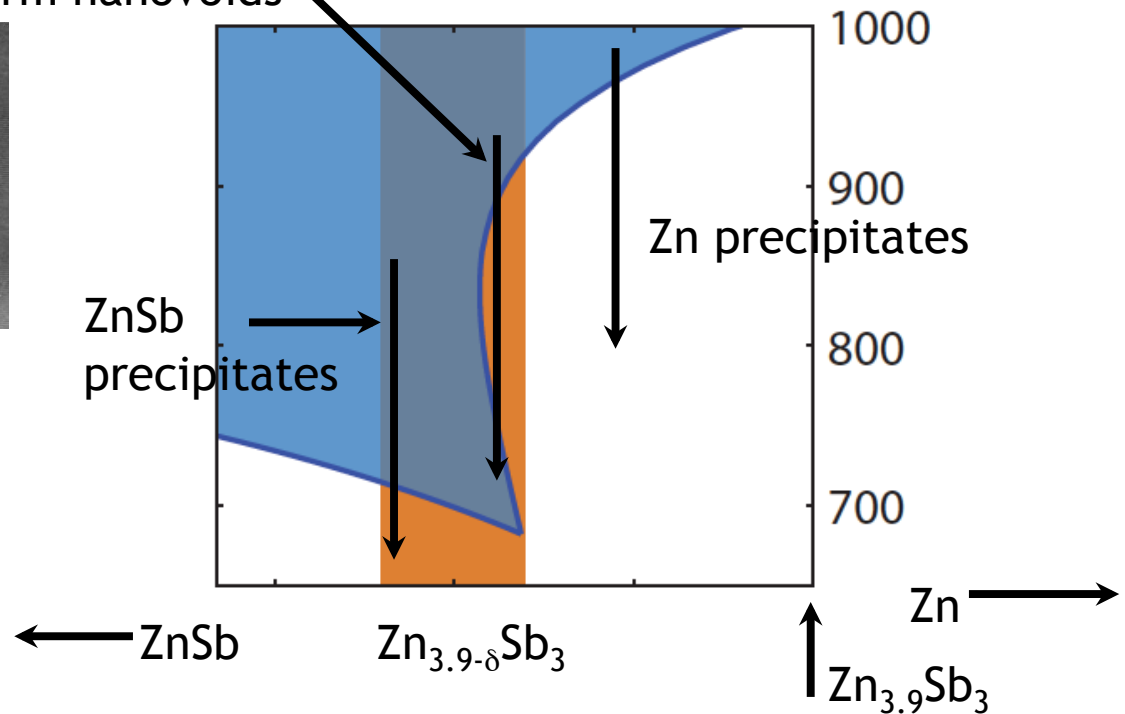
Pei et al Energy Env. Sci (2010) *in press*

Retrograde Solubility in Zn_4Sb_3



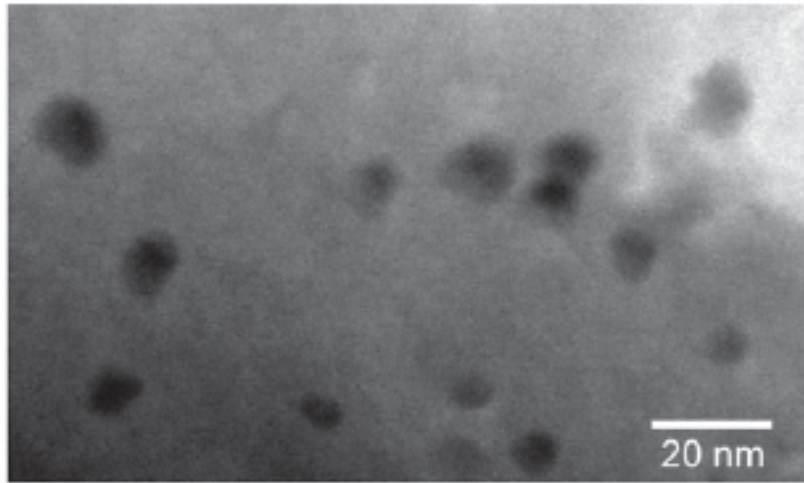
nanometer voids found in some samples of Zn_4Sb_3

Zn precipitate - reabsorbed to form nanovoids

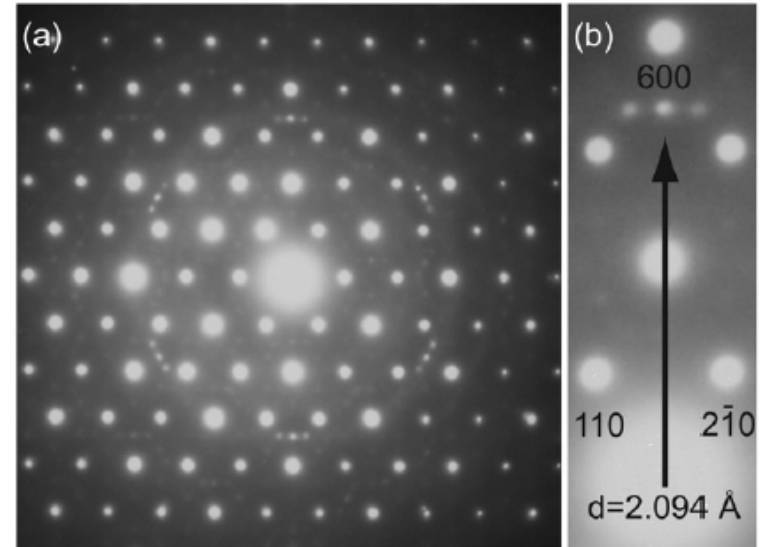


- Calculated phase diagram Zn_xSb_3 *ab initio*
- Confirms Zn_4Sb_3 stable due to configurational and Vibrational Entropy
- Correctly predicts phase should be Zn poor = *p*-type

Zn Precipitates in Zn_4Sb_3



- Coherent Zn precipitates found in some samples of Zn_4Sb_3



Fundamental material parameters

$$zT = \frac{\alpha^2 \sigma T}{\kappa} \rightarrow \beta = \frac{\mu m^{*3/2}}{\kappa_L}$$

Conflicts: Jeff Snyder's talk: $\mu = \frac{e\tau}{m^*}$

This talk: $\kappa_L \gg \kappa_{glass}$

How do we find materials which have $\kappa_L \approx \kappa_{glass}$?

‘Simple’ example - PbTe thermal conductivity

PbTe known to have low κ_L - why?

Sources of low κ_L :

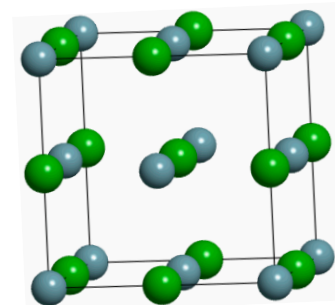
- Large Gruneisen parameter (1.5)
- High density
- Soft bonding

$v = \text{XXXX (l) and YYyy (t) m/s}$

Increased Umklapp scattering

Low v_s

No structural complexity
2 atoms in primitive cell



Outline

$$\kappa_L = \frac{1}{3} \int C v_g^2 \tau d\omega$$

Group velocity (v_g)

- Heavy atoms, soft bonding
 - PbTe
- Structural complexity
 - Yb₁₄MnSb₁₁
 - Ca₃AlSb₃

Scattering (τ)

- Intrinsic point defects
 - La_{3-x}Te₄
 - Zn₄Sb₃
- Nanostructures
 - Zn₄Sb₃