# Notes on the Economics of Household Energy Consumption and Technology Choice


**Alan H. Sanstad***
**Lawrence Berkeley National Laboratory**



**Abstract**

The Office of Management and Budget (OMB) and the Department of Energy (DOE) have initiated a joint effort to examine the issue of consumer welfare impacts of appliance energy efficiency standards, and to extend and discuss enhancements to the methodology by which these impacts are defined and estimated in the regulatory process. DOE's economic analysis of efficiency standards generally takes a life-cycle cost investment perspective focused on the trade-off between initial and operating costs for efficient equipment. In this perspective, the time value-of-money is represented by the cost of capital. In a more general framework, additional trade-offs exist between investment and consumption, and consumer choice over the planning horizon also reflects preferences for future consumption. In this framework, these preferences combine with the cost-of-capital as drivers of consumer choice. This document presents a first version of a mathematical framework for analyzing the similarities and differences between these two decision modeling approaches, and thus starts to address several theoretical economic issues raised by OMB. It is anticipated that further elaboration of this framework may support empirical analysis to develop practical quantitative tools for improved assessment of the effects of appliance standards.



* Staff Scientist, Lawrence Berkeley National Laboratory. #1 Cyclotron Rd., Berkeley, CA 94720. Voice: 510-486-6433; Fax: 510-486-6996; e-mail: ahsanstad@lbl.gov.

## 1. Introduction

U. S. federal appliance energy efficiency standards are established using a set of criteria pertaining to their effects on industry, consumers, environmental quality, and other factors. These notes are part of a joint effort by the Office of Management and Budget (OMB) and the Department of Energy (DOE) to examine the issue of consumer welfare impacts of efficiency standards, and to extend and enhance the methodology by which these impacts are defined and estimated in the regulatory process.

DOE's economic analysis of efficiency standards generally takes a life-cycle cost investment perspective focused on the trade-off between initial and operating costs for efficient equipment. In this perspective, the time value-of-money is represented by the cost of capital. In a more general framework, additional trade-offs may exist between investment and consumption, and consumer choice over the planning horizon can reflect preferences for future consumption. In this framework, these preferences combine with the cost-of-capital as drivers of consumer choice. This document presents a first version of a mathematical framework for analyzing the similarities and differences between these two choice modeling approaches, and thus starts to address several theoretical economic issues raised by OMB. These notes have been prepared to facilitate discussion and investigation of analytical metrics for assessing welfare effects, initially from a theoretical perspective. Terminology and basic concepts in engineering and economic approaches to modeling household or consumer energy demand are reviewed, and a simple theoretical economic model of consumer energy efficiency and fuel choice is introduced and discussed.

Going forward, this theoretical material may be useful in supporting empirical analysis to define and implement quantitative welfare estimates that relate life-cycle cost and other aspects of consumer appliance choices. This document reflects the philosophy that a clearly-articulated theoretical framework can be useful in dealing with the potential challenges and complexities of identifying and obtaining data for such estimates and integrating it into practical quantitative tools.

There is a long history of debate regarding consumer welfare effects of appliance standards, but the literature on this debate, as such, is not reviewed here. Indeed, this first version does not explicitly discuss standards *per se*. Instead, its aim is to facilitate discussion of the issues, provide a modeling starting point that can be discussed, debated, extended and improved, and to inform subsequent quantitative analysis. The departure point is the specific topic of metrics for assessing these consumer welfare effects. In the appliance standards regulatory methodology, direct consumer impacts are estimated by projecting life-cycle cost changes resulting from standards. In this paradigm, these direct cost outcomes are implicitly the measure of welfare effects. The key economic inputs to these calculations are purchase prices of appliances, energy prices, and discount rates. By contrast, the conventional microeconomic conception of consumer welfare is based on models of consumer utility maximization; while such models can be constructed in applications to yield welfare metrics denominated in dollars, the factors underlying these metrics include, in addition to equipment and energy prices and discount rates, so-called “behavioral parameters,” such as substitution elasticities, that empirically characterize consumers’ choices of energy and technology as predicted by utility maximization.

For applied quantitative analysis, all of these inputs and parameters – costs, prices, elasticities, etc. – must be empirically measured and/ or estimated, and issues such as data availability and quality, measurement error, and functional forms, must be addressed. The philosophy reflected in this first version of these notes is that these issues can tend to obscure the role of underlying principles; moreover, without a clear statement of theoretical and modeling assumptions, practical choices made in applied work – such as of functional forms for consumer utility - can have substantive consequences that may not be readily apparent (or desirable). In the present project, examples are the roles of utility discounting and intertemporal substitution *per se* in understanding consumer energy choices, as opposed to the appropriate numerical values of the rate-of-time-preference or substitution elasticity. In addition, a clear theoretical foundation can be invaluable in focusing and facilitating applied analysis. For these reasons, this draft deals strictly with theory; it establishes terminology and basic background concepts, and presents and discusses an initial simple theoretical model of consumer energy efficiency choice.

This is a working document in the sense that it is expected to evolve via revisions and additions from the stakeholders as the joint project proceeds. The exposition is intended to be self-contained (although familiarity with elementary optimization theory is assumed), and therefore deliberately begins from basic first principles of both engineering and microeconomic approaches to analyzing efficiency standards from the standpoint of consumer choice. For this reason, stakeholders are very likely to find some of the content already familiar – particularly Section 2. However, it is hoped that this summary of background material will help to clearly identify underlying assumptions and to define a frame of reference for analyzing consumer efficiency choices incorporating both engineering and economic perspectives and techniques.

## 2. Basic engineering and economic choice models

### 2.a The life-cycle cost model

Appliance efficiency standards are based in part on the observation that households derive value or utility not from the direct consumption of fuels – electricity, national gas, etc. – but rather from the *energy services* that are produced when these fuels are used in conjunction with energy-using equipment such as refrigerators, air-conditioners, and water heaters. Thus, refrigeration, air-conditioning, and water heating are examples of energy services. Within a given end-use energy service category, different unit models require different levels of fuel input to produce a given level of energy service, that is, have different engineering energy efficiencies.

Therefore, in principle, a specific energy service output level can be produced by different combinations of fuel and equipment; put differently, there exists a fuel-efficiency trade-off. Appliance standards act on this relationship by requiring a specific minimum efficiency – or equivalently, all else being equal, a maximum fuel demand - for a given equipment type and service level.

The regulatory process assumes that, in terms of prices faced by households in retail markets, the fuel-efficiency trade-off canonically corresponds to a cost trade-off, with more efficient equipment being more expensive to purchase initially while less expensive to operate. When combined with the assumption that both the level and the characteristics of the underlying energy service are held constant across fuel/ efficiency combinations, and initial costs, operating characteristics, and future fuel prices are assumed known with certainty, the problem of minimizing the cost of obtaining energy services is quite naturally captured in a deterministic discrete-time engineering-economic, i.e., discounted cash flow or life-cycle cost (LCC) model.[1]

In the case of a choice between two discrete efficiency levels, the LCC model is as follows. Suppose that for a given end-use energy service category, two units of equipment, labeled 1 and 2 respectively, have initial costs (purchase prices) $P_1$ and $P_2$ and require energy (fuel) inputs of $E_1$ and $E_2$ per period to produce an exogenously-given service level, with $P_1 < P_2$ and $E_1 > E_2$ . An example would be two refrigerators of equal volumes and features (i.e., of the same product class), one with higher purchase price but lower annual energy consumption in kilowatt hours under equivalent operating conditions. Further assume that the two units have the same anticipated operating lifetime of $T$ periods, that an initial fuel price $p_0$ is given and that a future sequence of per-period fuel prices is assumed, $p_1, p_2, \ldots$. Finally, assume that a fixed per-period "discount rate" $r$ is given. Then the expected LCC of purchasing and operating unit $i$, $i = 1, 2$, is

$$LCC_i = P_i + \sum_{t=0}^{T} \frac{p_t E_i}{(1+r)^t}, \tag{2.1}$$

where $p_t E_i$ is the operating cost of unit $i$ in period $t$. In this set-up, cost minimization means simply choosing the unit with the lowest LCC. Denoting the operating cost of the $i^{th}$ unit in period $t$ as $OC_{it} \equiv p_t E_i$, this criterion can be stated as: Choose the more efficient unit (#2) if and only if $LCC_2 < LCC_1$, i.e.,

[1] The choice of discrete rather than continuous time for this exposition reflects the convention used in the regulatory process and in much of the investment literature; it makes no substantive difference to the results.

$$P_2 + \sum_{t=0}^{T} \frac{OC_{2t}}{(1+r)^t} < P_1 + \sum_{t=0}^{T} \frac{OC_{1t}}{(1+r)^t}, \quad (2.2)$$

or

$$P_2 - P_1 < \sum_{t=0}^{T} \frac{OC_{1t} - OC_{2t}}{(1+r)^t}. \quad (2.3)$$

That is, the more efficient unit should be chosen if and only if the present value of the operating cost savings exceeds the incremental purchase price. This frames the energy efficiency choice as an investment decision using the net-present value criterion: The more efficient unit should be chosen if the initial "investment" of $P_2 - P_1$ is exceeded by the discounted sum of the stream of "returns" $OC_{1t} - OC_{2t}$. This highlights the critical role of the discount rate $r$. To explore this point further, assume that the fuel price is constant across periods, $p_t = p$ for all $t$, so that the (undiscounted) operating costs are also constant across periods, $OC_{it} = OC_i$ for all $t$. Recalling the summation formula

$$\sum_{t=0}^{T} \frac{1}{(1+r)^t} = \frac{1}{r}\left(1-(1+r)^{-T}\right), \quad (2.4)$$

and letting $\Delta P = P_2 - P_1$ and $\Delta OC = OC_1 - OC_2$, the previous LCC inequality can be re-arranged as

$$\frac{\Delta OC}{\Delta P} > r\left(\frac{1}{1-(1+r)^{-T}}\right). \quad (2.5)$$

In keeping with the investment interpretation, the expression $\frac{\Delta OC}{\Delta P}$ is the incremental return, in operating cost savings, for an incremental initial investment in energy efficiency. The right-hand side of this inequality approximates the discount rate $r$. Although an infinite horizon in this type of model is not practically relevant, it is nonetheless instructive – for mathematical simplicity – to imagine the limiting case as $T$ increases. In the limit, the inequality approaches

$$\frac{\Delta OC}{\Delta P} > r, \quad (2.6)$$

and equality – i.e., equal LCCs of the two technologies – would hold if $\Delta OC/\Delta P = r$. In other words, in this limiting case, the ratio $\Delta OC/\Delta P$ is the internal rate-of-return (IRR) of the efficiency investment, and the previous inequality is the standard criterion "invest if the IRR exceeds the discount rate."

Beyond this two-technology example, in practice – e.g., in appliance standards regulatory analysis - a set of technologies is usually posited, ranked by efficiency, with the purchase prices increasing, and operating costs decreasing, with higher efficiency – the familiar "cost curve." Notwithstanding the fundamentally discrete nature of technology types or options, it will be useful to consider a continuous generalization (or abstraction), which we introduce here and consider in more detail in the next section. Suppose that, for a given end-use energy service category, efficiency is represented by a continuous variable $\varepsilon$, the purchase price of technologies is represented by an increasing function $P(\varepsilon)$, and the per-period energy input requirement by a decreasing function $E(\varepsilon)$, where as in the above discrete example an exogenous service level is assumed. Then the operating cost in period $t$ is $OC_t(\varepsilon) = p_t E(\varepsilon)$, and we can write the life-cycle cost as a function of $\varepsilon$ as

$$LCC(\varepsilon) = P(\varepsilon) + \sum_{t=0}^{T} \frac{p_t E(\varepsilon)}{(1+r)^t}. \tag{2.7}$$

Further assuming differentiability of $P(\varepsilon)$ and $E(\varepsilon)$, the condition for life-cycle cost minimization is

$$\frac{dLCC}{d\varepsilon} = 0, \tag{2.8}$$

or

$$\frac{dP}{d\varepsilon} = -\sum_{t=0}^{T} \frac{p_t \, dE/d\varepsilon}{(1+r)^t}. \tag{2.9}$$

As before, if we assume constant energy price $p$ and therefore constant operating cost $OC(\varepsilon)$, then (using the summation formula above) the equation becomes

$$-\frac{dOC}{d\varepsilon} \Big/ \frac{dP}{d\varepsilon} = r\left(\frac{1}{1-(1+r)^{-T}}\right), \tag{2.10}$$

a continuous analogue to the discrete, two-technology condition $\Delta OC/\Delta P = r$.

### 2.b Static utility maximization

In this section, the basic two-good microeconomic model of consumer choice is briefly reviewed. In complete textbook expositions of this theory (for example, Varian 1992), a number of technical properties of utility functions and consumer optimization are defined and analyzed; here we note only the following essentials, in a two-good model. A hypothetical consumer is assumed to choose strictly positive quantities of goods or services $x_1$ and $x_2$ with prices $p_1$ and $p_2$, respectively, by maximizing a utility function $U(x_1, x_2)$ subject to the budget constraint that total expenditures on the two goods not exceed an amount $y$. The function $U(\cdot,\cdot)$ represents the consumers' preferences regarding $x_1$ and $x_2$ individually and jointly, and is assumed to be strictly increasing, twice continuously differentiable, and strictly concave in its arguments; in terms of partial derivatives, for $i = 1,2$,

$$\begin{gathered} \frac{\partial U}{\partial x_i} > 0, \\ \frac{\partial^2 U}{\partial {x_i}^2} < 0 \\ \frac{\partial^2 U}{\partial {x_1}^2}\frac{\partial^2 U}{\partial {x_2}^2} > \frac{\partial^2 U}{\partial x_1 \partial x_2}. \end{gathered} \tag{2.11}$$

The derivative $\partial U / \partial x_i$ is known as "marginal utility." The first inequality states that marginal utility is positive, i.e., utility increases with increased consumption. The second inequality states the property of "diminishing marginal utility," i.e., although utility increases, it does so at a declining rate.

We assume that the consumer possesses perfect information on these goods, services, and prices, and solves the choice problem under conditions of certainty. This terminology warrants clarification: In this context, "imperfect information" and "uncertainty" are distinct although related concepts. A standard example of decision-making under uncertainty is of choice among lotteries – i.e., bets – with outcomes defined by random variables the probability distributions of which are known to the consumer. In this case, the usual assumption is that the expected value of utility is maximized. This differs from the colloquial idea of having "imperfect information" in the sense of being uninformed or perhaps simply mistaken.

Mathematically, the consumer's behavior is described as solving the following optimization problem (where "max" means "maximize," and "s. t." means "subject to"):

$$\begin{gathered} max\, U(x_1, x_2) \\ s.t.\; p_1 x_1 + p_2 x_2 \leq y \end{gathered} \tag{2.12}$$

The Lagrangian for this problem is

$$\mathrm{L}(x_1, x_2, \lambda) = U(x_1, x_2) + \lambda(y - p_1 x_1 - p_2 x_2), \tag{2.13}$$

and the first-order necessary conditions for optimality are

$$
\begin{aligned}
\frac{\partial U}{\partial x_1} &= \lambda p_1 \\
\frac{\partial U}{\partial x_2} &= \lambda p_2 \qquad (2.14)\\
p_1 x_1 + p_2 x_2 &\leq y.
\end{aligned}
$$

Solving the first two equations for $\lambda$, equating the results, and re-arranging yields the condition

$$
\frac{\partial U}{\partial x_1} \Bigg/ \frac{\partial U}{\partial x_2} = p_1 / p_2 \ . \qquad (2.15)
$$

To gain some intuition for this, first suppose that $p_1 = p_2 = 1$. Then the condition is simply

$$
\frac{\partial U}{\partial x_1} = \frac{\partial U}{\partial x_2}, \qquad (2.16)
$$

that is, a necessary condition for utility maximization is that the levels of consumption of each of the two goods are such that marginal increases in utility from marginal increases in their consumption are equal. Were this not the case – say $\frac{\partial U}{\partial x_1} > \frac{\partial U}{\partial x_2}$ - then an infinitesimal increase in the consumption of $x_1$ and offsetting decrease in the consumption of $x_2$ would increase utility while staying within the budget constraint, contradicting the assumption of optimality. In the general case of $p_1 \neq p_2$, this logic is amended to take into account the unequal marginal costs, i.e., prices, of consuming an extra amount of either good, so that the ratio of marginal utilities at optimality equals the relative prices. It can be shown that what this in turn implies for the optimal relative quantities of $x_1$ and $x_2$ that are consumer depends on the concavity assumption; an example is discussed in detail in the next section.

## 3. A simple model of intertemporal energy service choice

### 3.a Modeling context

The previous section introduced the concept of energy services, such as refrigeration, air conditioning, and water heating, which are produced by combining energy (fuel) with energy-using equipment such as household appliances. The term “produced” is used here deliberately, for it places household energy and efficiency choices in the context of the economic theory of “household production” (Deaton and Muellbauer 1980, LaFrance 2001). This theory builds on an analogy to the theory of the firm by observing that households implicitly “produce” their own goods and services by combining various “raw” inputs, as in the present example, and that in a microeconomic sense these goods and services are the actual inputs to utility. The model presented below adopts this approach. The key goal is to explicitly address the household's energy service choice, rather than specifying it exogenously as in the life-cycle cost approach.

In the previous section, we described the life-cycle cost minimization problem in the case of two discrete technologies of differing efficiencies. Modeling consumer choice among a set of discrete options is the focus of what is known as "qualitative choice analysis" (QCA), a methodology which has been applied to energy technology choice in many studies. (See for example Train 1986.) Although QCA is in certain ways the natural approach to studying this type of consumer choice, it is also probabilistic and data-intensive, and designed specifically for numerical econometric analysis. Because these notes are initially concerned with underlying principles independent of statistical issues or the role of specific functional forms or parameter values, the following discussion instead assumes continuous relationships among energy efficiency, technology cost, and energy service, elaborating on the assumptions of the continuous life-cycle cost minimization model described in the previous section.

### 3.b Model assumptions

We assume that energy $E$ is an input to a technology that produces an energy service $S$ according to the relationship $S = E\varepsilon$, where $\varepsilon$ is a continuous index of service output per energy input. Thus $\varepsilon$ represents the technology's technical efficiency: Increasing $\varepsilon$ corresponds to an increased service output per unit of energy input or, equivalently, a reduced energy input requirement per unit of service. The cost of purchasing the technology with efficiency level $\varepsilon$ is assumed to be a twice continuously-differentiable function $P(\varepsilon)$ defined for $\varepsilon$ in the interval $E = [0, \varepsilon_{max}]$ (where $\varepsilon_{max}$ is a maximum-feasible efficiency level) and satisfying $P(0) = 0$, $P(\varepsilon) > 0$ for $\varepsilon > 0$, $\frac{dP}{d\varepsilon} \geq 0$, and $\frac{dP}{d\varepsilon} > 0$, $\frac{d^2P}{d\varepsilon^2} > 0$ for $\varepsilon > 0$, i.e., $P(\varepsilon)$ is positive, strictly increasing, and strictly convex for positive $\varepsilon$.

The basic static utility maximization model sketched in Section 2 can be extended in various ways to represent "dynamic" or "intertemporal" choice problems, i.e., those that involve a time dimension in the sense that decisions regarding future as well as present consumption are made, or planned, jointly. Such extensions can include time horizons of arbitrary length, for example the lifetime $T$ of energy-using equipment in the LCC model. In order to introduce as simply as possible the essentials of intertemporal choice from a microeconomic perspective as they apply to the energy and technology choice problem, however, we confine ourselves here to a two-period model in which the "present" corresponds to period 1, and "future" to period 2. We simplify further by initially focusing only on the intertemporal choice of energy services, rather than that of energy services as well as other household goods. Finally, we assume that the household or consumer has perfect information not only about present-period but also future prices and energy technologies, and makes the intertemporal choice under conditions of certainty. In the context of intertemporal choice, of course, these assumptions can be considered much stronger than their static analogues (as in Section 2b), for here they imply perfect foresight. Quantities of energy and energy services in periods 1 and 2 will be denoted $E_1, S_1$ and $E_2, S_2$, respectively.

The behavioral assumption is discounted utility maximization. This has a different meaning than the discounting of cash flows in the LCC model (for example); here, it means that anticipated future utility is given less "weight" than present utility. We assume that two-period utility is additive separable, and introduce an intra-period, single-good utility function $\mathrm{U}(\cdot)$ so that, in terms of the notation of the previous section, we can write

$$U(S_1, S_2) \equiv (S_1) + \frac{\mathrm{U}(S_2)}{(1+\rho)}, \tag{3.1}$$

where $\mathrm{U}(\cdot)$ is strictly increasing and concave - $\frac{d\mathrm{U}}{dS_i} > 0$, $\frac{d^2\mathrm{U}}{dS_i^2} < 0$ - and $\rho$ is a positive rate of time preference that represents the above-mentioned "weighting" of future utility.

We assume that an initial wealth endowment $W$, first period income $y_1$, and expected second period income $y_2$ are given exogenously. We also assume a fixed, exogenous "discount rate" $r$ that is a riskless rate for either borrowing or saving, implicitly through a capital market. One "dollar" in the present can be invested or saved to yield $1+r$ dollars in the future; conversely, one dollar can be borrowed in the present at the cost of repaying $1+r$ dollars in the future. The household's two-period budget constraint then takes the form

$$P(\varepsilon) + p_1 E_1 + \frac{p_2 E_2}{(1+r)} \leq W + y_1 + \frac{y_2}{(1+r)}, \tag{3.2}$$

which states that the present value of expenditures on fuel and technology must not exceed the present value of wealth and income. This present-value formulation derives from the existence of a capital market allowing the household to shift wealth or income, and therefore expenditures and consumption, between present and future. Note that this interpretation makes sense even if the discount rate is zero; the point is that the consumer is not restricted to spending wealth or income only in the period in which they become available. (Put differently, the budget constraint for the two-period problem does not consist of two single-period budget constraints.) Instead, in the first period, the household faces a choice of allocating between consumption and saving.

### 3.c Model solution and interpretation

The household chooses levels of energy services in periods 1 and 2 to solve the following problem:

$$\begin{aligned} &\max_{S_1,S_2} \ \mathrm{U}(S_1)+\frac{\mathrm{U}(S_2)}{(1+\rho)} \\ s.t.\ & P(\varepsilon)+p_1E_1+\frac{p_2E_2}{(1+r)} \le W+y_1+\frac{y_2}{(1+r)} \\ & S_1=E_1\varepsilon \\ & S_2=E_2\varepsilon. \end{aligned} \tag{3.3}$$

Note that in keeping with the perspective that energy services are the inputs to utility, the service demands in the two periods are the decision variables in this problem. However, this raises the following issue: While the equations relating services to energy can be used to substitute for the energy terms in the budget constraint, the definition of the optimization appears to leave unresolved the problem of choosing the optimal value of energy efficiency $\varepsilon$.

To deal with this, we use the household production theory idea of "two-stage" or "two-tiered" decision-making: In the lower tier, the optimal combination of fuel and efficiency is determined conditional on the allocation of service consumption between the two periods; at the top tier, this intertemporal allocation is decided using the lower-tier results as inputs. In this model, the lower-tier optimality criterion is cost-minimization. Under our assumptions, given values $S_1,S_2$ of services, the cost $C(S_1,S_2)$ of "producing" these services is the sum of fuel expenditure in period 1, discounted fuel expenditure in period 2, and the cost of the technology as a function of the efficiency level, that is:

$$C(S_1,S_2)=P(\varepsilon)+p_1E_1+\frac{p_2E_2}{(1+r)}, \tag{3.4}$$

where $E_1=\frac{S_1}{\varepsilon}$ and $E_2=\frac{S_2}{\varepsilon}$. The lower-tier problem is thus

$$\begin{aligned} &\min_{E_1,E_2,\varepsilon} P(\varepsilon)+p_1E_1+\frac{p_2E_2}{(1+r)} \\ s.t.\ & E_1\varepsilon=S_1 \\ & E_2\varepsilon=S_2. \end{aligned} \tag{3.5}$$

Note that this is a special case of the continuous version of life-cycle cost minimization described in Section 2, with the operating cost function implicitly defined here by $OC_t(\varepsilon)=p_t\,S_t/\varepsilon$. It is shown in the Appendix that the solution to this problem is a unique efficiency level $\varepsilon^*$, energy consumption quantities $E_1^* = S_1 / \varepsilon^*, E_2^* = S_2 / \varepsilon^*$, and Lagrange multipliers $\tau_1^*, \tau_2^*$.

The optimal values $\tau_1^*, \tau_2^*$ of the Lagrange multipliers in the cost-minimization problem are called the *shadow prices* of the energy services in the two periods. Under our assumptions, they are the marginal costs of the energy services. Returning to the two-stage framework, these shadow prices allow us to re-write the top-tier utility maximization problem as

$$\begin{gathered} \max_{S_1, S_2} \; \mathrm{U}(S_1) + \frac{\mathrm{U}(S_2)}{(1+\rho)} \\ s.t. \; \tau_1^* S_1 + \frac{\tau_2^* S_2}{(1+r)} \le W + y_1 + \frac{y_2}{(1+r)}. \end{gathered} \tag{3.6}$$

In other words, we have subsumed or embedded the solution to the conditional energy service cost minimization problem, and the top-tier problem is now to allocate energy service consumption intertemporally, i.e., between periods 1 and 2. The Lagrangian for this problem is

$$\mathrm{L}(S_1, S_2, \lambda) = \mathrm{U}(S_1) + \frac{\mathrm{U}(S_2)}{(1+\rho)} + \lambda \left( W + y_1 + \frac{y_2}{(1+r)} - \tau_1^* S_1 - \frac{\tau_2^* S_2}{(1+r)} \right) \tag{3.7}$$

and the first-order conditions are

$$\begin{gathered} \frac{d\mathrm{U}}{dS_1} = \lambda \tau_1^* \\ \frac{1}{(1+\rho)} \frac{d\mathrm{U}}{dS_2} = \lambda \frac{\tau_2^*}{(1+r)} \\ \tau_1^* S_1 + \frac{\tau_2^* S_2}{(1+r)} \le W + y_1 + \frac{y_2}{(1+r)}. \end{gathered} \tag{3.8}$$

As in the previous section, we can solve for $\lambda$ and combine the first two equations to obtain the condition

$$\frac{d\mathrm{U}}{dS_1} \Big/ \frac{d\mathrm{U}}{dS_2} = \frac{\tau_1^*}{\tau_2^*} \frac{(1+r)}{(1+\rho)}. \tag{3.9}$$

This equation describes the optimal trade-off – i.e., between present and future consumption of energy services – in this intertemporal problem. Comparing with Equation 2.15, we see that the static optimality relationship is now augmented by the introduction of the rates of discounting and time preference: In addition to the effect of the prices $\tau_1^*$ and $\tau_2^*$, the extent to which the utility-maximizing household will favor either present or future consumption is also influenced by the relative magnitudes of $r$ and $\rho$, as reflected in the ratio $\frac{1+r}{1+\rho}$.

We mentioned in the previous section the importance of concavity in interpreting the optimality condition in the static model. Analogously, in this two-period model the concavity of $\mathrm{U}$ plays a critical role, which we now describe. To explain this, suppose first that $r = \rho = 0$, i.e., that the household does not discount future consumption and that there is no cost for borrowing nor a "return" for saving. In this case, the "dynamic" problem is completely equivalent to the static; the optimality condition equates the ratio of marginal utilities to that of prices. Now consider the effect of incrementally increasing $r$ and $\rho$ so that both are positive, and re-optimizing. If $r > \rho$, then the right-hand side of Equation 3.9 will increase to maintain equality; if $r < \rho$, it will decrease. Thus, the relative magnitudes of $d\mathrm{U}/dS_1$ and $d\mathrm{U}/dS_2$ must in turn adjust. What does this imply for the adjustment in the consumption levels of $S_1$ and $S_2$? Because the concavity assumption implies that $d^2U/dS_i^2 < 0$, it follows that $dU/dS_i^2$ will increase (decrease) if and only if $S_i$ decreases (increases). Suppose that $r > \rho$; then to maintain optimality $d\mathrm{U}/dS_1$ must increase relative to $d\mathrm{U}/dS_2$, and therefore $S_1$ must decrease relative to $S_2$. (Either $S_1$ must decrease or $S_2$ must increase, or both.) Conversely, by exactly analogous reasoning, it follows that if $r < \rho$ after the incremental change, then $S_1$ must *increase* relative to $S_2$.

Let us interpret these conclusions in terms of the consumption/saving choice noted above. The consumer derives utility from the consumption of energy services, but must decide how to allocate this utility between present and future. As in the static problem, the relative period 1 and period 2 prices favor either present or future consumption. However, the assumption of a non-zero rate-of-time preference $\rho$ means that, all else being equal, the consumer prefers present to future consumption. Moreover, by virtue of the capital market, the consumer also has the option to borrow in order to increase present consumption at the expense of future, or to save and thereby have additional resources to finance consumption in the future. The discussion in the previous paragraph says that the consumption vs. saving dimension of this choice problem is governed by the ratio $\frac{1+r}{1+\rho}$. If $r > \rho$, then the value of marginal incremental future consumption available from deferring present consumption in order to save and subsequently increase $S_2$ exceeds the loss of utility from lower consumption of $S_1$. Conversely, if $r < \rho$, then at the margin present consumption is favored.

**3.d Discussion**

We now summarize the main elements of this section. We introduced a deterministic two-period model of energy service choice by a discounted utility maximizing consumer subject to an intertemporal budget constraint. Adopting a two-stage budgeting approach from household production theory, we showed that the consumer's optimal solution incorporates life-cycle cost minimization conditional upon two-period energy service demands. The roles of the rates of discount and time preference (in addition to prices) in determining the consumer's allocation of energy service consumption between present and future were discussed, including the importance of concavity in the utility function.

With respect to the background and objectives presented in the Introduction, we would emphasize the following points. First, in this simple model, life-cycle cost minimization and intertemporal utility maximization are not alternative assumptions, but consistent and inter-related; this result should be robust to at least some extensions (including multi-period time horizons). Second, the distinction between the "discount rate" and the rate of time preference is critical, and highlights the problems of using only observed or implicit discount or "hurdle" rates in an engineering-economic framework to make inferences about consumers' intertemporal choices related to energy efficiency and technology. Although the details will vary, this conclusion will also be robust in more complex models (including those applying different behavioral assumptions than exponential discounting of utility). Third, even this simple model begins to indicate the type and importance of microeconomic "behavioral parameters," such as the rate of time preference.

### 3.e Elaborating the model

As stated in the Introduction, these notes have been prepared as part of a joint OMB-DOE discussion on consumer welfare effects of appliance efficiency standards, and are aimed at facilitating discussion of issues and contributing to further work on technical methodology. Such work can proceed in several (complementary) directions. Specifically with regard to theoretical extensions of the model and results described above, next steps could include the following. First, in addition to the rate-of-time-preference, the so-called "intertemporal elasticity of substitution" is a critical behavioral parameter in dynamic choice models, and its effects on optimal choice in the model should be analyzed. Second, to investigate how energy service choice is combined with, and affected by, the consumer's preferences for other goods and services, a non-energy composite good can be introduced. Third, extensions to longer time horizons are important – in particular, the lifetime of energy technology as represented in the life-cycle cost model. Fourth, even before considering empirical applications, it will be important to introduce specific functional forms for utility in order to gain understanding of how these may affect the results. Fifth, more complex representations of energy service "production technologies" – e.g., incorporating possible nonlinearities – should be explored. As these examples indicate, there are multiple possibilities for further development.

## 5. References


Deaton, Angus, and John Muellbauer. 1980. *Economics and consumer behavior*. Cambridge, England: Cambridge University Press.

LaFrance, Jeffrey T. 2001. "Duality for the Household: Theory and Application." Chapter 18 in B. Gardner and G. Rausser, eds., *Handbook of Agricultural Economics.* New York: Elsevier Science.

Train, Kenneth E. 1986. *Qualitative Choice Analysis: Theory, Econometrics, and an Application to Automobile Demand*. Cambridge, Massachusetts: The MIT Press.

Varian, Hal R. 1992. *Microeconomic Analysis*. New York: W. W. Norton.

**Appendix**

The existence of a solution to the cost-minimization problem in Section 3 can be shown as follows. The Lagrangian for this problem is

$$\mathrm{L}\left(E_1,E_2,\varepsilon,\tau_1,\tau_2\right)=P\left(\varepsilon\right)+p_1E_1+\frac{p_2E_2}{\left(1+r\right)}+\tau_1\left(S_1-E_1\varepsilon\right)+\tau_2\left(S_2-E_2\varepsilon\right)$$

and the first-order optimality conditions are

$$\begin{gathered}\frac{dP}{d\varepsilon}-\tau_1E_1-\tau_2E_2=0\\ p_1-\tau_1\varepsilon=0\\ \frac{p_2}{\left(1+r\right)}-\tau_2\varepsilon=0\\ S_1=E_1\varepsilon\\ S_2=E_2\varepsilon.\end{gathered}$$

This is a system of five equations in five unknowns; by first solving for $E_1,E_2$ in the fourth and fifth equations and then for $\tau_1,\tau_2$ in the second and third, we can reduce the system to a single equation in the unknown $\varepsilon$:

$$\frac{dP}{d\varepsilon}\varepsilon^2=p_1S_1+\frac{p_2S_2}{\left(1+r\right)}. \qquad (*)$$

Because we have deliberately avoided introducing specific functional forms, we cannot solve this equation explicitly for $\varepsilon$; instead, we reason as follows. We assume (trivially) that $p_1S_1+\frac{p_2S_2}{\left(1+r\right)}>0$, and by the assumptions on $P\left(\varepsilon\right)$, $\left.\frac{dP}{d\varepsilon}\right|_{\varepsilon=0}=0$. Differentiating,

$$\frac{d}{d\varepsilon}\left[\frac{dP}{d\varepsilon}\varepsilon^2\right]=\frac{d^2P}{d\varepsilon^2}\varepsilon^2+2\frac{dP}{d\varepsilon}\varepsilon>0$$

for $\varepsilon$ > 0, where the inequality again follows from the assumptions on $P\left(\varepsilon\right)$ (strictly increasing and concave, with continuous second derivative). If we make the reasonable assumption that the quantity $p_1S_1+\frac{p_2S_2}{\left(1+r\right)}$ lies in the range of $P\left(\varepsilon\right)$, it follows that there is a unique solution $\varepsilon^*$ to Equation $*$ (by the Intermediate Value Theorem and the strict monotonicity of $P\left(\varepsilon\right)$).